State of the Art on X-Ray CT Reconstruction Methods

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Abstract – **The goal of this paper is to give a short overview to the complex field of x-ray CT reconstruction methods, focusing on the main classes of reconstruction methods and on some of the methods actually in use in modern x-ray CT systems.**

I. INTRODUCTION

Since the origins of x-ray CT (due to the pioneering work of Hounsfield and Kormack) to present time, tomographic reconstruction has been one of the most dynamic research topics of the last forty years. An impressive number of scientific publications and meetings has been devoted to this research field and the trend is likely to continue.

The goal of this paper is to give an overview to the complex research field of x-ray CT reconstruction, focusing on the main classes of reconstruction methods and on some of the methods actually in use in modern x-ray CT systems.

The extraordinary evolution of x-ray CT is based on the interconnected and reciprocally challenging developments in systems' technology, mathematical methods and new clinical applications. Because of this we won't consider the reconstruction methods in a purely theoretical framework, instead, we'll constantly refer practical aspects like the specific system geometry, acquisition protocol or clinical application to which the different methods are suitable.

We'll start with some basic aspects and with the formal definition of the mathematical problem of tomographic reconstruction in the original 2D geometrical framework (section II).

In section III we shortly describe the two main algorithms for 2D tomographic reconstruction, direct Fourier (DF) and filtered backprojection (FBP) methods. Most of the state of the art 2/3D reconstruction methods belong to the idea of FBP algorithm.

With the introduction of spiral acquisition geometry and multi-slice detectors, the acquisition/reconstruction processes need to be described in a 3D framework. Nevertheless, under certain conditions, CT reconstruction methods continue to follow a 2D paradigm. In section III, we review the reconstruction methods suitable for multislice spiral x-ray CT.

II. BASICS OF X-RAY CT RECONSTRUCTION

The distribution of the attenuation coefficient on a transversal section of an object can be described as a 2D function *f* (object function) in the *(x,y)* plane of the section.

The two parameters θ and *s* univocally specify the line with equation

$$x\cos\theta + y\sin\theta = s \quad (1)$$

in the *(x,y)* plane and the general formula for the line integral, known as the Radon transform of *f(x,y)*, is:

$$p(\theta,s) = \iint f(x,y)\delta(x\cos\theta + y\sin\theta - s)dxdy \quad (2).$$

The purpose of x-ray CT 2D reconstruction methods is to calculate *f(x,y)* given a proper set of measured line integrals, that means, from a mathematical point of view, to calculate the inverse Radon transform given a sufficient set of samples.

A projection consists of a collection of integrated values of *f(x,y)* taken along a set of straight lines in the plane and the projection data set is given by a number of projections taken with different orientations. Basically, two geometries have been defined for the sets of line integrals making a 2D projection: parallel and divergent (or fan-beam).

In parallel geometry (the acquisition geometry of first generation systems, shown in figure 1), a projection $p_\theta(s)$ consists of a collection of line integrals taken along straight parallel lines in the plane, that means a collection of $p(\theta,s)$ with constant θ and $s \in [-S/2, S/2]$. A parallel projection data set is usually represented as a 2D matrix, called sinogram, each row of which corresponds to a value for the parameter θ (a parallel projection) and each column to a value for the parameter *s*.

In divergent geometry (the acquisition geometry of third generation systems, shown in figure 2) to each angular position of the focal spot corresponds a fan of focus-detector lines (the detector being an array of detector

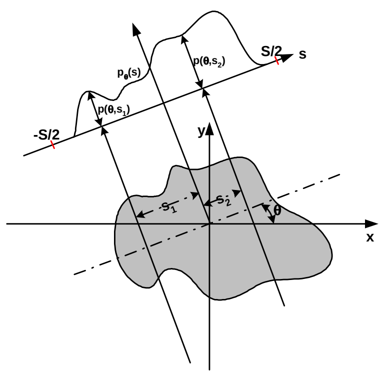

Fig. 1 Object function *f(x,y)* and its parallel projection in θ direction.

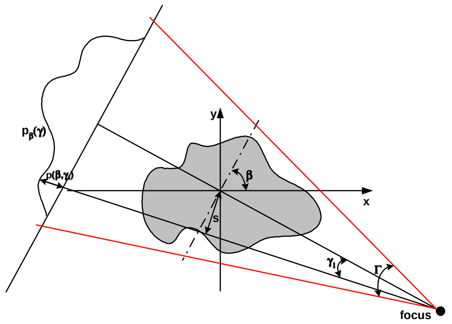

Fig. 2 Object function *f(x,y)* and its divergent projection in β direction.

elements). Given the ray of focal spot trajectory, the parameters β and γ define the line of equation

$$x\cos(\beta+\gamma)+y\sin(\beta+\gamma)=-r\sin\gamma$$

and each divergent projection is a collection of line integrals taken with constant β and $\gamma\in[-\Gamma/2,\Gamma/2]$.

In Radon space, the space of the Radon transform of *f(x,y)*, a projection data set for a given geometry corresponds to a set of samples taken over a specific sampling grid. As a consequence, given a set of samples obtained with a given geometry a different sample set can be calculated by interpolation. This transformation, called rebinning, is commonly performed to obtain a parallel projection sample set given a divergent one.

A fundamental result in tomographic reconstruction is the Fourier slice theorem (details and demonstration can be found in [1]):

Theorem 1: The Fourier transform of a parallel projection of an object function *f(x,y)* taken at angle θ gives a slice of the two-dimensional Fourier transform of *f(x,y)*, *F(u,v)*, subtending an angle θ with the *u* axis.

In other words, the 1D Fourier transform $P_\theta(\sigma)$ of the parallel projection $p_\theta(s)$, gives the values of *F(u,v)* along line BB in figure 3.

III. 2D X-RAY CT RECONSTRUCTION

The Fourier slice theorem suggests a simple way to solve the reconstruction problem. Taking parallel projections of the object function *f* at angles θ_1, θ_2 .. θ_n and Fourier transforming each of them, we obtain the 2D Fourier transform of the object function *F(u,v)* on *n* radial lines. In ideal conditions (infinite number of projections and samples per projection) *F(u,v)* would be known at all

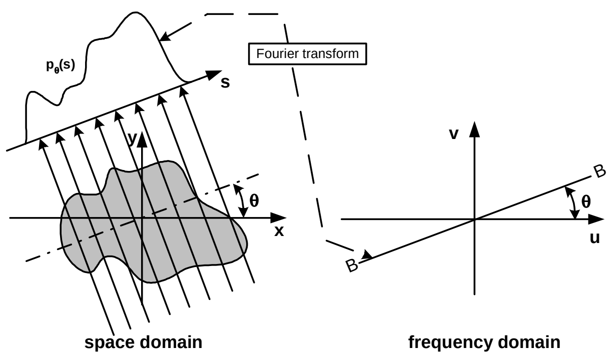

Fig. 3 Graphic representation of Fourier slice theorem statement.

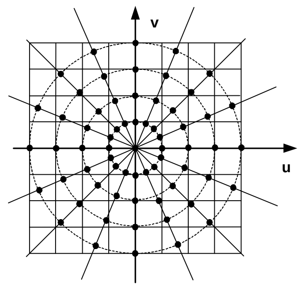

Fig. 4 In Fourier reconstruction, sample points of the 2D Fourier transform of the image are given on a conventional polar grid.

points in the frequency domain and the object function *f(x,y)* could be recovered by 2D inverse Fourier transforming *F(u,v)*.

Fourier reconstruction methods (also known as direct Fourier or Fourier based methods) follow directly from this ideal procedure, adapted to the discrete case [2]. Since only a finite number of projections and samples per projection are taken, *F(u,v)* is known just on a finite number of points along a finite number of radial lines (fig. 4) and, in order to obtain an approximation of *f(x,y)* by 2D inverse Fourier transform of *F(u,v)*, first we have to interpolate from the radial points to the points on a Cartesian grid.

Unfortunately, the results obtained with such a straight method suffer from artefacts due to interpolation in Fourier space and aliasing. Nevertheless, due to its low computational complexity ($O(N^2 \log N)$) this method has been object of research and various techniques have been proposed in order to improve its performance. Some techniques are based on peculiar sampling schemes –polar interleaved grid [3], polar squared grid [2], linogram [4]– while some others take advantage on recent development in the calculation of NUFFT (Non Uniform Fast Fourier Transform) [5], allowing for the 2D inverse Fourier transform of *F(u,v)* directly from its radial samples. Especially these last methods have shown a performance equivalent to the one of the more widely accepted FBP algorithm with a considerable saving in computational time. The fact that, in principle, these algorithms are not suitable for reconstruction from divergent projections doesn't seem to be an obstacle anymore since, in any case, 2D reconstruction is performed on interpolated data (after longitudinal interpolation), being possible to choose a parallel geometry resampling grid. Moreover, it has been demonstrated that, taking advantage of NUFFT, direct Fourier methods can be applied directly on divergent projections [6].

For historical and practical reasons, the most successful 2D reconstruction method (chosen by all the manufacturers) is FBP. The reconstruction formula can be mathematically derived [1], but the method can also be introduced in a very intuitive fashion.

If we smear back (backproject) the measured samples along the direction with which they were taken, we obtain a blurred version of the image we were supposed to get.

This problem is solved by filtering (with a ramp filter) the projections before backprojecting them. This method can be applied to divergent geometry by adding additional weighting to the projections and in the backprojection process.

Despite their higher computational complexity ($O(N^3)$), FBP methods have been preferred over DF methods because they offer the possibility to perform the acquisition and reconstruction processes at the same time, to adjust image quality by choosing different (harder or softer) filters, to be easily extended to new acquisition geometries. Nowadays, since reconstruction is always performed "a posteriori" and the NFFT based DF methods also need a filtering step, giving the possibility to choose between softer and harder filters, we can say that (in 2D reconstruction) FBP and DF methods have equivalent performance.

In most applications it's important to limit as much as possible the number of projections to consider in the reconstruction of an image. In case of divergent projections we just need a set of projections corresponding to a focal spot rotation arc of $\pi+\Gamma$ (being Γ the fan angle), called short scan data set [7]. Both FBP and DF algorithms can be modified in order to perform reconstruction based on short scan data sets.

Moreover, it has been demonstrated that an object point can be reconstructed exactly if it "sees" a scan path segment of angular range π. Thus, if we are interested in the reconstruction in just a limited ROI, an even smaller data set is sufficient (super short scan). Specific reconstruction algorithms have been developed to reconstruct ROI images based on a super short scan data set [8, 9].

IV. SPIRAL CT: FROM 2 TO 3D RECONSTRUCTION

With the introduction of multislice spiral acquisition, although the acquisition geometry is defined in the 3D space, up to a small number of slices (tipically 8) image reconstruction is still managed as a 2D reconstruction problem. The spiral acquisition geometry is defined introducing a new parameter, called pitch, which is the ratio of table movement per rotation and collimator aperture.

An additional processing step called longitudinal interpolation allows for the synthesis of a consistent planar data set from the spiral data for an arbitrary image

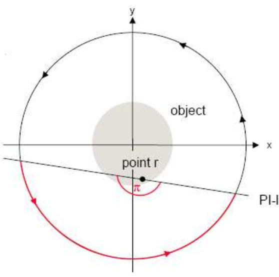

Fig. 5 Focal spot rotation arc corresponding to a super short scan data set.

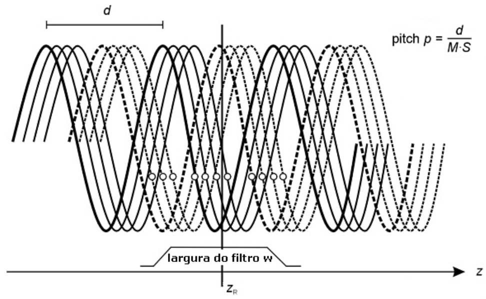

Fig. 6 Interpolation scheme in 180ºFMI longitudinal interpolation algorithm [10].

position, then, 2D reconstruction is performed. The possibility to retrospectively select image position and reconstruction increment provides the most significant advantage of spiral acquisition.

Various different approaches to longitudinal interpolation have been proposed: 360º/180ºLI –Linear Interpolation– (for single slice acquisition), 360º/180ºMLI –Multislice Linear Interpolation– and 180ºMFI – Multislice Filtered Interpolation– (for multislice acquisition). In figure 6 the interpolation scheme of the 180ºFMI longitudinal interpolation algorithm is represented. In this algorithm are selected for interpolation (filtered) all the samples corresponding to a longitudinal window of pre-defined width. Slice thickness can be altered by setting different filter widths [10].

For higher number of slices (wider cone angle), the error associated with longitudinal interpolation cannot be neglected anymore. A number of algorithms have been developed taking into account the focal spot trajectory and the cone angle of different slices. Basically, in these algorithms 2D reconstruction is performed over tilted image planes (fig. 7), then, the multiple tilted images are interpolated (filtrated) to obtain axial images. The tilted

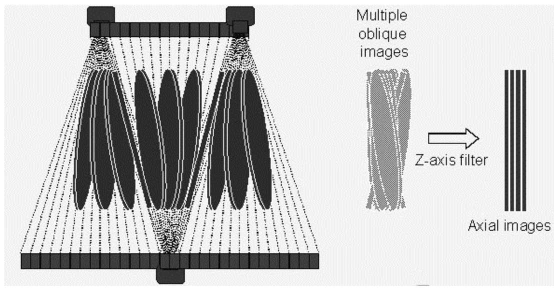

Fig. 7 2D reconstruction over tilted planes and longitudinal flter to obtain axial images (www.impactscan.org).

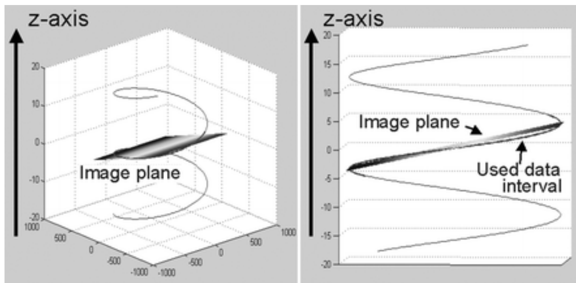

Fig. 8 In ASSR algoritm the image plane is selected in order to minimize the reconstruction error [11].

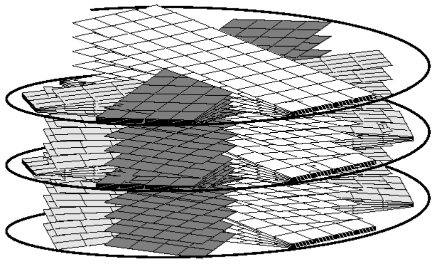

Fig. 9 Reconstrução de imagens parciais in SMPR algorithm [14].

planes are selected in order to better fit the segment of trajectory used for the reconstruction. Techniques like ASSR –Advanced Single Slice Rebinning– and AMPR –Adapted Multiple Plane Reconstruction– follow this strategy [12, 13].

In the SMPR –Segmented Multiple Plane Reconstruction– technique [14], short segments of focal spot trajectory are considered. For each of these segments the data corresponding to each detector slice are rebinned to a small set of parallel projections, which are filtered and backprojected to the image plane that better fits the focal spot trajectory and the cone angle of the detector slice, obtaining a set of incomplete images on planes with different inclination (fig. 9). Finally, in order to obtain an axial image, the partial images have to be longitudinally interpolated and combined. This or similar strategies are used in some modern 64 slices CT systems (Siemens, GE).

Another possible reconstruction strategy, already followed by manufacturers like Philips and Toshiba in their state of the art multi slice CT systems, is fully 3D reconstruction. These algorithms are extension of the Feldkamp algorithm, an approximate 3D filtered backprojection algorithm developed for sequential scanning [15], to multislice spiral scanning. With this approach, the measurement rays are first weighted and filtered (similarly to what happen in 2D filtered backprojection for divergent geometry) and then backprojected into a 3D volume along the lines of measurement, accounting in this way for their cone-beam geometry. Examples of Feldkamp type 3D filtered backprojection algorithms for spiral trajectory are [16-18]. Three-dimensional backprojection has a high computational complexity ($O(N^4)$) and requires dedicated hardware to reduce image-reconstruction times.

CONCLUSIONS

We presented a short overview on x-ray CT reconstruction methods. Although 2D acquisition geometries are almost outdated, 2D reconstruction is still in use since, due to the high computational complexity of 3D reconstruction algorithms, most of the reconstruction methods continue to follow one of the paradigms:

- sinogram synthesis -> 2D axial plane reconstruction
- 2D tilted plane reconstruction -> axial plane image interpolation.

Due to space constraints, we have limited our analysis to the most important methods.

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