A New Process to Measure the Jitter of a System

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***Resumo*-Este trabalho apresenta um novo processo para medir o jitter, causado por ruído, num sistema de comunicações.**

Primeiro mostramos alguns medidores de jitter convencionais e depois apresntamos o novo processo proposto. Testaremos o novo medidor de jitter na PLL (Phase Lock Loop) analógica.

O objectivo é certificar o novo processo, mostrando que os resultados obtidos com ele são coerentes com os valores teóricos e experimentais, disponíveis para a PLL analógica. Mostramos o jitter de saída UIRMS (Unit Interval Root Mean Squared) como função da entrada SNR (Signal to Noise Rate).

***Abstract* - This work presents a new process to measure the signal jitter, caused by noise, in a communication system.**

First, we show some conventional jitter measurers and after we present the new proposed process. We will test the new jitter measurer on the analog PLL (Phase Lock Loop).

The objective is to certify the new process, showing that the results obtained with it are coherent with the theoretical and experimental values, available for the analog PLL. We show the output jitter UIRMS (Unit Interval Root Mean Squared) as function of the input SNR (Signal to Noise Rate).

I. INTRODUCTION

We present a new process to measure the jitter of a communication system. We need to guarantee that our propose is exact and precise. Thus, we need firstly to test it on the well known analog PLL whose jitter theoretical formula σ_ϕ^2=NoBl/A^2 and jitter experimental values are available [1]. We will prove that our results are coherents.

There are various jitter measurers, we show the more traditionals and after, we add the new process. We compare our jitter measure propose with the traditionals.

Fig.1 shows the setup, to measure the jitter of a system under test, in this case the analog PLL.

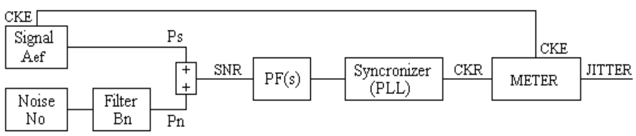

Fig.1 Setup to measure the jitter of a system

This setup gives the system output jitter as function of the input noise. If these measured results are equal to theoretical and experimental values, then the new process is credible. So, it can be used with fidelity, in future measurements.

II. TRADITIONAL JITTER MEASURERS

We present firstly some traditional jitter measurers, all with Kd=1, for understand better the new proposed process.

All the jitter measurers have phase detectors, whose output characteristic curve is function of the input phase variation ϕ.

The output characteristic curve (dynamic range) must be so ideal as possible, with maximum linear proportional zone and maximum monotone crescent zone, for the input range.

The filter, after the phase detector, passes the desired low frequencies and attenuates strongly the undesired high frequencies. The detector gain Kf multiplied by the gain K is equal to Kd. Following, we show the phase detectors, with their output characteristic curves f(ϕ) as functions of the input phase variation ϕ, in range 2π [0, 2π] or 4π [-2π, 2π].

A. Ideal multiplier with filter

Fig.2 shows the ideal multiplier as base of the jitter measurer. We can see its output characteristic curve y(ϕ) as function of the inputs phase variation (difference) $\phi \in [0, 2\pi]$.

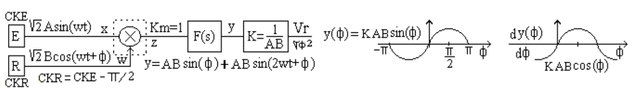

Fig.2 Ideal multiplier with filter

The output z(t) can be expressed in terms of the two inputs (x(t), w(t)) and the phase difference ϕ

$$z(t) = x(t) \cdot w(t)$$
$$= \sqrt{2}A\sin(\omega 1t+\phi 1) \cdot \sqrt{2}B\cos(\omega 1t+\phi 2)$$
$$= AB\sin(\phi 1-\phi 2) + AB\sin(2\omega 1t+\phi 1+\phi 2)$$

The first term is selected (low attenuation) and the second term is eliminated (great attenuation). So

$y(\phi) = AB\sin(\phi 1-\phi 2)$ considering $\phi 2=0$ we have
$y(\phi) = AB\sin\phi 1 = AB\sin\phi$

We can see that there is no linear zone and is monotone crescent only in the interval $\pi = [-\pi/2, \pi/2]$.

For little phase variations, we can consider $\sin\phi \cong \phi$, so
$y(\phi) = AB\phi$

The slope at the origin is $dy(t)/dt=Kf=AB$, so $K=1/AB$. The medium point of the monotone crescent zone or stable equilibrium point SEP=0V. So, to sum any offset is not need. So, with a reduced linear zone $\sin\phi \cong \phi$ and with a monotone crescent zone π, we can only measure correctly low jitter amplitudes. Thus, we will choose others measurers.

B. Exor with filter

Fig.3 shows the Exor gate as base of the jitter measurer. We can see its output characteristic curve $y(\phi)$ as function of the inputs phase variation (input dynamic range) $\phi \in [0, 2\pi]$.

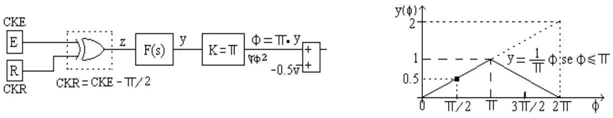

Fig.3 Exor with filter

The output $y(\phi)$ is expressed in terms of the inputs (E,R), the detector gain $Kf=1/\pi$ and the phase difference ϕ. The slope at the origin is $Kf=1/\pi$. So, $K=\pi$ and therefore $Kd=Kf.K=1$. The stable equilibrium point is SEP=0.5V. Then, for DC=0, the correction offset to sum is 0.5V.

Thus, we see the output characteristic curve has a linear proportional zone of 2π and a monotone crescent zone of $\pi = [0, \pi]$ for input excursion $2\pi=[0, 2\pi]$. As result we can only measure jitter correctly with amplitudes less than π.

C. RS flip flop with filter

Fig.4 shows the RS flip-flop as base of the jitter measurer. We can see its output characteristic curve $y(\phi)$ as function of the inputs phase variation (input dynamic range) $\phi \in [0, 2\pi]$.

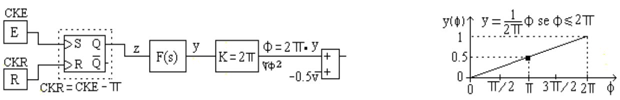

Fig.4 RS flip flop with filter

The output $y(\phi)$ is expressed in terms of the inputs (E,R), detector gain $Kf=1/2\pi$ and phase difference ϕ. The slope at the origin is $Kf= 1/2\pi$. So, $K=2\pi$ and therefore $Kd=Kf.K=1$. The stable equilibrium point is SEP=0.5V. Then, for DC=0, the correction offset to sum is -0.5V.

Thus we see the output characteristic curve has a linear proportional zone of 2π and a monotone crescent zone of $2\pi = [0, 2\pi]$ for input excursion $2\pi=[0, 2\pi]$. As result now, we can correctly measure jitter with amplitudes up to 2π.

Fig.5 shows that, with an operation of high duty-cycle (50%) pulse error, there is some spectral components of the high term that lays inside of the filter bandwidth and measured as jitter that slightly falsifies the results.

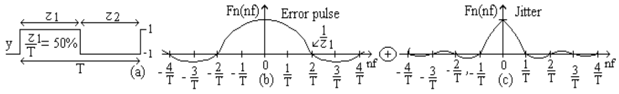

Fig.5 There is significant spectral lines at low frequency

Then, we can reduce the spectral power inside the bandpass of the filter, diminishing the pulse error duty cycle as happens in the next jitter measurer (2 D flip flops and AND).

D. Two D flip flops and AND with filter

Fig.6 shows the two flip flops and AND with reset as base of the jitter measurer. We can see its output characteristic curve $y(\phi)$ as function of the inputs phase variation (input dynamic range) $\phi \in [-2\pi, 2\pi] = 4\pi$.

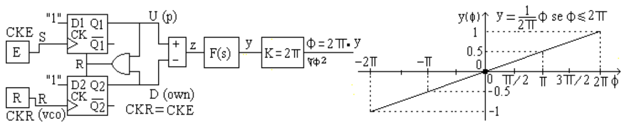

Fig.6 Two D flip flops and AND with filter

The output $y(\phi)$ is expressed in terms of the inputs (E,R), detector gain $Kf=1/2\pi$ and phase difference ϕ. The slope at the origin is $Kf= 1/2\pi$. So, $K=2\pi$ and therefore $Kd=Kf.K=1$.

The stable equilibrium point is SEP=0V. Thus, for DC=0, it isn't necessary to sum any offset.

Thus, we see the output characteristic curve has a linear proportional zone of 4π and a monotone crescent zone of $4\pi =[-2\pi, 2\pi]$ for input excursion $2\pi=[0, 2\pi]$. As result now, we can correctly measure jitter with amplitudes up to 4π.

Fig.7 shows that, with an operation of low duty cycle (0%) pulse error, there is a shift of great quantity of energize to high frequencies, which easily can be eliminated by the filter, without falsifying the results.

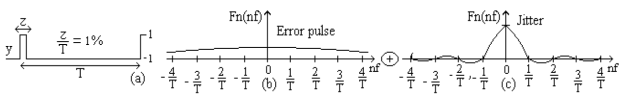

Fig.7 There is a shift of spectral lines for high frequencies

The RS flip flop characteristic curve is $y(\phi)=(1/2\pi)\phi$ then the inverse will be $\phi=2\pi y(\phi) <=> \phi=360y(\phi)$.

This equation is valid for the flip flop RS and for the two D flip flops and AND.

III. THE NEW PROPOSED JITTER MEASURER

Our propose substitutes the filter for a converter (ideal filter) that passes the low frequencies and eliminates the high ones. This makes the results more rigorous.

A. New method: RS flip flop with converter

Fig.8 shows the RS flip flop with converter (sampling and holder) as base of the jitter measurer. We can see its output characteristic curve $y(\phi)$ as function of the input $\phi \in [0, 2\pi]$.

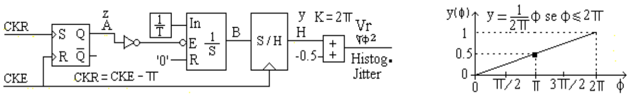

Fig.8 RS flip flop with converter

The output $y(\phi)$ is function of the inputs (E,R), detector gain $Kf=1/2\pi$ and phase ϕ. The slope is $Kf=1/2\pi$. So, $K=2\pi$ and then $Kd=Kf.K=1$. The stable equilibrium point is SEP=0.5V. Then, for null DC, it is necessary to sum -0.5V.

The output curve has a linear zone of 2π and monotone zone of $2\pi=[0, 2\pi]$. We can correctly measure jitter with amplitudes up to 2π. This measurer has a sampling and hold that functions as ideal filter eliminating completely the high frequencies error pulse without attenuating the jitter.

Fig.9 illustrates the operation of the jitter measurer based on the RS flip flop with a converter of phase variation into amplitude variation, which is the jitter histogram.

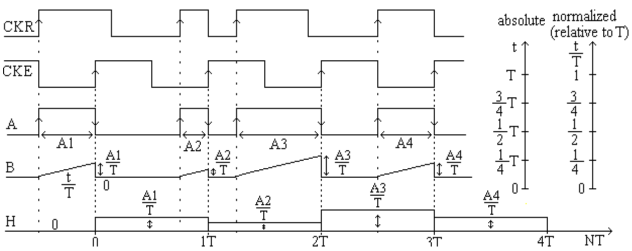

Fig.9 Waveforms at the RS flip flop with converter

The jitter histogram is then sampled and after processed by an appropriate program giving the average m, the jitter variance σ_ϕ^2, the jitter standard deviation UIRMS and the jitter standard deviation peak to peak UIPP. We used sequences of 10000 bits but only the last 9000 are processed. The sampling period of 10^{-3} implies a ratio of 1000 samples per bit, but only one from 100 to 100 is processed.

The program, which processes the jitter histogram, was initially written in C language and after in Matlab language.

```
s=size(y);
n=s(1);
m=mean(y)
b=y-m;
c=sumsqr(b);
v2=c/n;
vr2=vv2*4*pi^2
dpr=sqrt(vr2);
dpui=dpr/(2*pi)
dpv=sqrt(v2);
TP=1;
dpu=dpv/TP;
A=max(y);
B=min(y);
dpuipp=abs(A-B)/TP;
```

IV. TESTS, DESIGN AND RESULTS

In order to validate the new jitter measurer, it is fundamental to prove it on the well known analog PLL[2, 3].

A. The known analog PLL used as certificate test

The analog PLL, based on the ideal multiplier, was used as guarantee certificate, since its theoretical formula $\sigma_\phi^2=NoBl/A^2$ and tabled experimental values are available in literature [1] and can be compared with the measured results obtained with the new jitter measure (Fig.10).

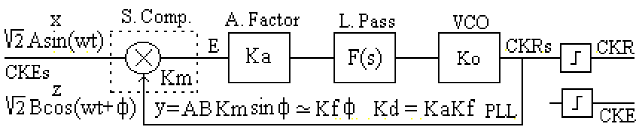

Fig.10 Analog Phase Lock Loop

This PLL (Phase Lock Loop) has a phase comparator Kf, a loop filter F(s) and a VCO (Voltage Controlled Oscillator) Ko and an amplification factor Ka that controls the roots.

B. Sampling and precision

The analog PLL signals must be processed digitally by the computer then it is need previously to sample the signals.

Fig.11 shows the sampling process in which the spectral noise density No is related with the noise standard deviation σ_n and with the sampling period $\Delta\tau$.

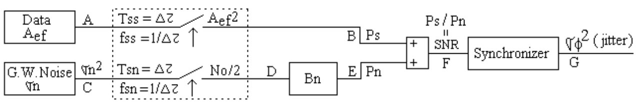

Fig.11 Sampling process

The spectral power density No/2 is the power inside 1Hz. Fig.12 relates No with the noise variance σ_n^2 and with the sampling period $\Delta\tau$. Considering a noise constant source

No/2 passing through the bandwidth filter $1/2\Delta\tau$, then No/2 will be equal σn^2 multiplied by $\Delta\tau$ which gives $No=2\Delta\tau\sigma n^2$.

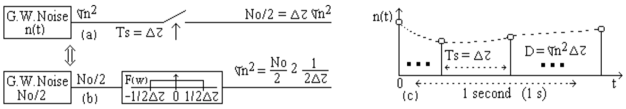

Fig.12 Illustration of the power spectral density

Fig.13 shows the relation between No and σn that depends on the considered case: to be unilateral or bilateral.

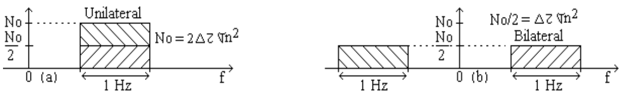

Fig.13 Unilateral case (a) and bilateral case (b) (factor 2)

The spectral noise density for unilateral case (positive) is $D(w)=No=2\Delta\tau\sigma_n^2$ and for bilateral case (positive and negative) is $D(w)=No/2=\sigma_n^2\Delta\tau \rightarrow No=2\Delta\tau\sigma_n^2$.

We consider the bilateral case and the jitter-noise formula is $\sigma_\phi^2=NoBl/A^2= 2\Delta\tau\sigma_n^2Bl/A^2$ as the following dimensioning.

C. Loop parameters dimensioning

The synchronizer is designed to operate in the linear zone for which the theoretical formula is approached.

For correct comparisons all the loops must be designed with the same linearized transfer functions.

The jitter measurer, after the calibration with the analog PLL, can be used to measure the jitter of general systems.

The PLL parameters (F(s), Kf, Ko) are pre-fixed but the amplification factor Ka, is conveniently adjusted to control the root locus and then the desired loop characteristics.

Following, we present the 1st and 2nd order loop dimensioning of the analog PLL. In both cases, we consider normalized values for the operation frequency fo=1Hz, external noise bandwidth Bn=5Hz and loop noise bandwidth Bl=0.02Hz.

- 1st order loop

In the 1st order loop, the filter F(s) does not influence the loop characteristics, but only eliminates the high frequency perturbation terms, produced by the phase detector. So F(s)=0.5Hz is 25 times greater than Bl=0.02Hz.

For this loop the transfer function is

$$H(s)=\frac{G(s)}{1+G(s)}=\frac{KdKo}{s+KdKo} \quad (1)$$

and the loop noise bandwidth is

$$Bl=\frac{KdKo}{4}=Ka\frac{KfKo}{4}=0.02Hz \quad (2)$$

For the analog synchronizer Km=1, A=B=1/2, Ko=2π, then

$$Ka\frac{KmABKo}{4}=0.02Hz \quad \Rightarrow \quad Ka=0.08\frac{2}{\pi} \quad (3)$$

- 2nd order loop

with $F(s)=\dfrac{1+sT2}{sT1}$ the transfer function is (4)

$$H(s)=\frac{sKdKo(T2/T1)+KdKo/T1}{s+sKdKo(T2/T1)+KdKo/T1} \quad (5)$$

$$=\frac{sA+B}{s^2+s2\xi Wn+Wn^2} \quad (6)$$

and the loop noise bandwidth is

$$Bl=\frac{\xi Wn}{2}\left(1+\frac{1}{4\xi^2}\right) \quad (7)$$

Taking (ξ=1, Bl=0.02 and Kd=1/2π) and solving the above equations, we obtain for F(s)

$$F(s)=\frac{1+s63}{s977} \quad (8)$$

So we have

$$Kd=KaKf=Ka(1)(1/2)(1/2)=\frac{1}{2\pi}\rightarrow Ka=\frac{2}{\pi} \quad (9)$$

In the tests, we opted by the 1st order loop and we obtained the following results.

D. Results

Tab.1 shows the measured output jitter (variance Vrs2 and standard deviation dpui2) as function of the input noise standard deviation σ_n. We also present others theoretical intermediate values such as (Spectral Noise Density No, Signal Noise Ratio SNR5, Jitter Variance Vrt2) also as function of the input noise σ_n. Then we can relate this data.

Tab.1 Output jitter versus input noise standard deviation

	Input	Theoretical Intermediate Values			Outputs (Simulated)	
*	σ	No	SNR5	Vrt2	Vrs2=Φr^2	Dgui2
*	Param.	$\Delta\tau=10^{-3}$ No= $2\Delta\tau\sigma n^2$	Bn=5 SNR5= A^2/NoBn	Bl=0.02 Vrt2= NoBl/A^2	Bn=5 Bl=0.02 F(s)=1	Bn=5 Bl=0.02 F(s)=1
0	0	0	∞	0	7.17e-6	4.26e-4
1	0.7906	0.0013	40	0.0001	1.14e-4	0.0017
2	1.1180	0.0025	20	0.0002	2.16e-4	0.0023
3	1.25	0.0031	16	0.00025	2.66e-4	0.0026
4	1.5811	0.0050	10	0.0004	4.19e-4	0.0033
5	1.7678	0.0063	8	0.0005	5.24e-4	0.0036
6	2.5	0.0125	4	0.001	0.0010	0.0051
7	3.5355	0.0250	2	0.002	0.0021	0.0072
8	5	0.0500	1	0.004	0.0041	0.0102
9	5.5902	0.0625	0.8	0.005	0.0051	0.0114
10	7.9057	0.1250	0.4	0.01	0.0103	0.0161
11	11.1803	0.2500	0.2	0.02	0.0206	0.0228
12	17.6777	0.6250	0.08	0.05	0.0521	0.0363
13	25	1.2500	0.04	0.1	0.1064	0.0519
14	35.3553	2.5	0.02	0.2	0.2268	0.0758
15	50	5	0.01	0.4	0.5500	0.1180
16	61.2372	7.5	0.007	0.6	0.9716	0.1569

Fig.14 relates graphically the measured (simulated) jitter variance σ_ϕ^2 (vrs2) obtained with the new measurer as function of the jitter variance theoretical formula σ_ϕ^2=vrt2=NoBl/A^2.

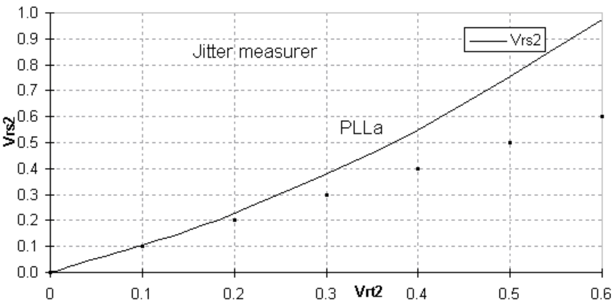

Fig.14 Measured RMS jitter versus theoretical RMS jitter

The graphic shows that for input theoretical jitter quantities Vrt2<0.1, the measured (simulated) results vrs2 with the new process are identical to the theoretical and experimental results. However, for input theoretical jitter quantities Vrt2>0.1, the measured results with the new technique continues to be identical to the experimental results obtained by Rosenkranz [1] but begins to be slightly different from the theoretical results, due to the formula linear approach.

V. CONCLUSIONS

To guarantee the proposed new jitter measurer, we tested its calibration on the well known analog PLL.

The PLL was designed to operate in the linear mode, then only for low noise quantities, the jitter-noise behavior is described by the theoretical formula $\sigma\phi^2$=NoBl/A^2. Thus, in this case, the results measured by the new proposed process are according to the theoretical and experimental results.

However, for higher noise quantities, the analog PLL can no longer be considered in the linear mode, then the above theoretical formula is only an approach. Anyway, the results measured by the new process continue identical to the ones obtained experimentally by Rosenkranz [1], but begins to have a slight difference with the theoretical results.

So, the results measured with the new process are identical to the experimental values for low and high noise quantities and similar to the theoretical values for low noise quantities.

Then, we conclude that the new proposed technique is credible, since its measured (simulated) results are coherent with the experimental and theoretical values. So, we can use this technique, in the future, with great reliability.

ACKNOWLEDGMENTS

The authors are grateful to the FCT (Foundation for sCience and Technology).

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