Chromatic dispersion monitoring technique using optical asynchronous sampling and double sideband filtering

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Abstract **– We present the equations that led to the novel final equation, which relates chromatic dispersion with the power amplitude, measured in a time window.**

Keywords **– Chromatic dispersion, optical sampling**

I. INTRODUCTION

Chromatic dispersion is well known as one of the most limiting impairments in optical communications. Prior work in the field of chromatic dispersion monitoring, implemented various schemes, including asynchronous amplitude histograms [1], [2], RF tone measurement [3], [4], self phase modulation(SPM), four wave mixing(FWM) and cross phase modulation(XPM) [5]-[7], polarization scrambling [8], asynchronous chirp [9], two photon absorption(TPA) with semiconductor microcavity [10], and so on.
RF tone measurement techniques cannot isolate chromatic dispersion, because are known to be sensitive to a variety of distortion effects including PMD [11]. Although, relatively speaking ,is a technique with moderate dynamic range, cost, and acquisition time and also suitable to implement [12]. In this paper we expose the theory that led to a new expression for the chromatic dispersion in a optical fiber.

II. THEORY

The electric field equation of a sinusoidal input modulation, after passing through a dispersive fiber, an ideal filter and an interferometer is given by (1) [13]:

$$\begin{aligned} r(t) =& \sqrt{P_0} H_1 e^{i\phi_1 + 2\,i\pi\,t\,f_0} \\ & \left(1 + 1/4\,\frac{m\,(H_2 + H_3)\,e^{i\left(q\,{f_p}^2 - \phi_1 + 1/2\,\phi_2 + 1/2\,\phi_3\right)}}{H_1}\right. \\ & \frac{\cos(2\,\pi\,f_p\,t + 1/2\,\phi_2 - 1/2\,\phi_3)}{H_1} + \\ & 1/4i\,\frac{m\,(H_2 - H_3)\,e^{i\left(q\,{f_p}^2 - \phi_1 + 1/2\,\phi_2 + 1/2\,\phi_3\right)}}{H_1} \\ & \left.\frac{\sin(2\,\pi\,f_p\,t + 1/2\,\phi_2 - 1/2\,\phi_3)}{H_1}\right) \otimes \\ & \otimes\, \mathbb{F}^{-1}\{rect(f_0, f_{BW})\}, \end{aligned} \tag{1}$$

where f_p is the frequency of the sinusoid, f_0 is the carrier frequency, P_0 is the average laser launch power, m is the modulation index, $\mathbb{F}^{-1}\{rect(f_0, f_{BW})\}$ is the inverse Fourier transform of an ideal filter centered at f_0, with frequency bandwidth equal to f_{BW} and $\otimes$ is the convolution operator. The $H_1, H_2, H_3, \phi_1, \phi_2, \phi_3$ and q , parameters are defined as follows:

$$\begin{aligned} &H_1 = |\cos(\pi\tau f_0)| \\ &H_2 = |\cos(\pi\tau\,(f_0 + f_p))| \\ &H_3 = |\cos(\pi\tau\,(f_0 - f_p))| \\ &\phi_1 = -\pi\tau f_0 + \tfrac{\pi}{2} + \angle\cos(\pi\tau f_0) \\ &\phi_2 = -\pi\tau f_0 + \tfrac{\pi}{2} + \angle\cos(\pi\tau\,(f_0 + f_p)) - \pi\tau f_p \\ &\phi_3 = -\pi\tau f_0 + \tfrac{\pi}{2} + \angle\cos(\pi\tau\,(f_0 - f_p)) + \pi\tau f_p \end{aligned} \tag{2}$$

$$q = \frac{\pi\lambda_0^2 DL}{c}, \tag{3}$$

where λ_0 is the carrier central wavelength, D is the dispersion parameter, L is the fiber length, $\angle$ is the angle operator and τ is the delay time given by:

$$\tau = \frac{1}{2f_p}. \tag{4}$$

In such conditions we can write:

$$\begin{array}{cc} H_1 = |\cos(\pi\tau f_0)| & H_2 = |\cos(\pi\tau\,(f_0 + f_p))| \\ H_3 = H_2 & \phi_1 = 2\pi n + \alpha \\ \phi_2 = 2\pi n + \alpha + \frac{\pi}{2} & \phi_3 = 2\pi n + \alpha + \frac{\pi}{2}, \end{array} \tag{5}$$

where α is an arbitrary angle dependent of f_0 . The optical power is given by:

$$\begin{aligned}
P_{opt} =& |r(t)|^2 = \mathrm{Re}\,(r(t))^2 + \mathrm{Im}\,(r\,(t))^2 = \\
& \Big(1/4\,\sqrt{P_0} m\, H_2 \\
& \cos\left(2\,\pi\, t\, f_0 + q\, {f_p}^2 + \phi_2 + 2\,\pi\, f_p\, t\right) + \\
& 1/4\,\sqrt{P_0} m\, H_3 \\
& \cos\left(2\,\pi\, t\, f_0 + q\, {f_p}^2 - 2\,\pi\, f_p\, t + \phi_3\right) + \\
& \sqrt{P_0} H_1\, \cos\left(2\,\pi\, t\, f_0 + \phi_1\right)\Big)^2 + \\
& \Big(1/4\,\sqrt{P_0} m\, H_2 \\
& \sin\left(2\,\pi\, t\, f_0 + q\, {f_p}^2 + \phi_2 + 2\,\pi\, f_p\, t\right) + \\
& 1/4\,\sqrt{P_0} m\, H_3 \\
& \sin\left(2\,\pi\, t\, f_0 + q\, {f_p}^2 - 2\,\pi\, f_p\, t + \phi_3\right) + \\
& \sqrt{P_0} H_1\, \sin\left(2\,\pi\, t\, f_0 + \phi_1\right)\Big)^2 \\
=& \frac{1}{16} P_0 (m^2 {H_2}^2 + \\
& + 2m^2 H_2 H_3 \cos(\phi_2 + 4\pi f_p t - \phi_3) + \\
& + 8m H_1 H_2 \cos(q {f_p}^2 + \phi_2 + 2\pi f_p t - \phi_1) + \\
& m^2 {H_3}^2 + \\
& 8m H_3 H_1 \cos(q {f_p}^2 + \phi_3 - 2\pi f_p t - \phi_1) \\
& + 16 {H_1}^2).
\end{aligned} \tag{6}$$

Using (5) into (6) we obtain:

$$\begin{aligned}
P_{opt} = \frac{1}{16}\, P_0\, & \left({H_2}^2 m^2 + 2\, m^2 H_2\, H_3\, \cos\left(4\,\pi\, f_p\, t\right) - \right. \\
& 8\, m\, H_2 H_1\, \sin\left(q\, {f_p}^2 + 2\,\pi\, f_p\, t\right) + m^2 {H_3}^2 \\
& \left. -8\, m\, H_3\, H_1\, \sin\left(q\, {f_p}^2 - 2\,\pi\, f_p\, t\right) + 16\, {H_1}^2\right).
\end{aligned} \tag{7}$$

To calculate the maximum amplitude of the signal we must derivate (7) in order to time and equalize it to zero:

$$\begin{aligned}
\frac{dP_{opt}}{dt} =& P_0\, m\, H_3\, H_1\, \cos\left(q\, {f_p}^2 - 2\,\pi\, f_p\, t\right) f_p\, \pi - \\
& 1/2\, P_0\, m^2 H_3\, H_2\, \sin\left(4\,\pi\, f_p\, t\right) f_p\, \pi - \\
& P_0\, H_1\, m\, H_2\, \cos\left(q\, {f_p}^2 + 2\,\pi\, f_p\, t\right) f_p\, \pi = 0.
\end{aligned} \tag{8}$$

Then we must find the solutions that fulfil this requirement. We conclude that the solutions are:

$$t = \begin{cases} \left(2i+1\right)/\left(2 f_p\right) & if, \begin{array}{l} D\,L \geq 2n D_{Talbot} \\ DL < \left(2n+1\right) D_{Talbot} \end{array} \\ \left(2i+1\right)/\left(f_p\right) & if, \begin{array}{l} DL \geq \left(2n+1\right) D_{Talbot} \\ DL < \left(2n+2\right) D_{Talbot} \end{array} \end{cases} \tag{9}$$

where $n = ..., -2, -1, 0, 1, 2, ...$ and $i = 0, 1, 2, ...$.
The product DL is the total accumulated dispersion in the fiber. A more thorough study need to be done, because of the use of asynchronous sampling, but some clues can be found taking into account that:

$$T_s = \frac{1}{\frac{k}{j} f_p} \quad k < j, \tag{10}$$

where T_s is the sampling period. Then the following condition must be met:

$$\frac{k}{j} \geq \frac{1}{2m+1}, \tag{11}$$

where m is the i^{th} solution of (8) , that represents the solution with highest value in an appropriate time window. D_{Talbot} is defined as [14]:

$$D_{Talbot} = \frac{c}{{f_p}^2 {\lambda_0}^2}. \tag{12}$$

D_{Talbot} is due to the Talbot effect, which describes the apparent re-emergence of a periodic sequence of pulses, propagating in the dispersive medium.
Substituting the solutions given by (9) into (7), for instance for $i = 1$:

$$\begin{aligned}
P_{opt}\,(q) = \;& \tfrac{1}{16} P_0 \left(m^2 {H_2}^2 + 2m^2 H_2 H_3 + \right. \\
& +16 m H_2 H_1 \sin(q {f_p}^2) + m^2 {H_3}^2 \\
& +16 {H_1}^2
\end{aligned} \tag{13}$$

yields in (13), which is a novel relationship between the maximum amplitude of the signal and chromatic dispersion.
The ratio between the optical power and the average power is given by:

$$\begin{aligned}
ratio =& \frac{P_{opt}(q)}{P_{avg}} = \frac{P_{opt}(q)}{\frac{1}{16}\, P_0\, \left({H_2}^2 m^2 + m^2 {H_3}^2 + 16\, {H_1}^2\right)} = \\
=& \begin{cases} \left(1 + \dfrac{2m^2 H_2 H_3}{\left({H_2}^2 m^2 + m^2 {H_3}^2 + 16\, {H_1}^2\right)} + \right. \\ \left. + \dfrac{m H_2 H_1}{\left({H_2}^2 m^2 + m^2 {H_3}^2 + 16\, {H_1}^2\right)} sin\left(q {f_p}^2\right)\right) \\ \left(1 + \dfrac{2m^2 H_2 H_3}{\left({H_2}^2 m^2 + m^2 {H_3}^2 + 16\, {H_1}^2\right)} - \right. \\ \left. - \dfrac{16 m H_2 H_1}{\left({H_2}^2 m^2 + m^2 {H_3}^2 + 16\, {H_1}^2\right)} sin\left(q {f_p}^2\right)\right) \end{cases} = \\
=& 1 + \frac{2m^2 H_2 H_3}{\left({H_2}^2 m^2 + m^2 {H_3}^2 + 16\, {H_1}^2\right)} + \\
& \left| \frac{16 m H_2 H_1}{\left({H_2}^2 m^2 + m^2 {H_3}^2 + 16\, {H_1}^2\right)} sin\left(q {f_p}^2\right) \right|
\end{aligned} \tag{14}$$

which leads to an equation that is independent from the launch power P_0.

III. CONCLUSIONS

We show the theory for a new chromatic dispersion monitoring technique based in asynchronous optical sampling, that led to a new equation for the chromatic dispersion.

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