

# Analysis and design of a limit-cycle envelope amplifier for wireless transmitters

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**Abstract** – Limit-cycle or self-oscillating power amplifiers (SOPAs) are seen as a compact option to achieve efficient yet linear amplification of either complete RF signals or only their base-band envelopes. This paper summarizes the work carried out in the Master Thesis of the author, which consisted on the analysis and design of a limit-cycle amplifier, taking into account the propagation delay of the relay. For that, the theory of its working principle is discussed, and a prototype is built. During the work, a method to increase the self-oscillation frequency for a fixed propagation delay and loop filter was proposed, which consists on replacing the relay with one that exhibits a novel kind of hysteresis. To validate the concept, simulations are carried out to compare the behavior of existing types of SOPA with the proposed one.

**Keywords** – Limit-cycle amplifier, negative hysteresis, power amplifiers, relay control system.

## I. INTRODUCTION

Energy efficiency is a hot-topic nowadays. Regarding electronic systems, our world is now facing an unprecedented growth on the number of devices, increasing our global energy needs despite the fact that the energy has never been as expensive as now. Moreover, the ever-growing demands for performance also increase the energy requirements, being one of the main performance-limiting factors.

In the wireless communications industry, the power is also a very important factor. Not only on the mobile radios, but also on those belonging to the infrastructure (e.g. base-stations) which need to cope with high RF power. The main problem arises from the fact that when amplifying signals of modern modulations, with high Peak-to-Average Power Ratio (PAPR), the average efficiency of RF Power Amplifiers hardly reaches 40%, moreover, the power needed to cool down the RF apparatus reduces the overall efficiency even further. The increase in power efficiency have two advantages, in one hand the reduction of wasted power, on the other hand the possibility to run such base-stations off-grid using energy-harvested energy, one of the main aims of the so called “Green Radio” [1], [2].

The linearity-efficiency problem on signal amplification has long been studied, and several classes of amplification have been developed. A lot of attention have been drawn into switched RF stages [3] [4] [5], and the results show that efficiencies up to 80% can be obtained. However, these results only apply for constant-envelope signals, and when the PAPR is increased, the Average Power Added Efficiency is rapidly degraded. Moreover, it has been proven that the sum of constant-envelope uncorrelated signals also results in high PAPR signals [6]. Two major techniques have been developed in order to use such constant-envelope ampli-

fiers with large PAPR signals, both based on the concept of dynamically adjusting the power supply voltage. One technique, entitled Envelope Elimination and Restoration [7] removes the envelope information from the RF signal that feeds the RF amplifier, and the power-supply modulator, usually known as envelope amplifier, needs to amplify efficiently the envelope signal with minimum error. Envelope Tracking [8] is an alternative that does not remove the envelope information from the RF signal that feeds the RF amplifier, thus requiring less performance from the envelope amplifier. Regarding the envelope amplifier itself, several approaches have been reported in the literature, for example [8], [9], [10], [11], among others. One of the most interesting architectures, explored in [9] and [12], is the self-oscillation-based switched amplifiers. These amplifiers are very simple from the system’s point of view, yet, they are capable of efficient and linear amplification of signals, namely, the base-band envelope.

Unfortunately, although several analysis can already be found in the literature for this kind of systems, none of them considered the signal propagation delay, which constitutes a significant limitation for their use in envelope amplifiers intended to operate at higher switching frequencies, and thus to process nowadays broadband signals. Indeed, the objective of this work is to derive the necessary conditions for the oscillatory behavior, accounting for the propagation delay and hysteresis in order to predict the self-oscillating frequency, and the input-to-output quasi-static transfer characteristics. These are the two fundamental parameters needed to predict the SOPA’s switching losses (and thus efficiency), bandwidth and non-linear distortion behavior.

This paper is divided in five main sections. After this introduction, Section II gives some insight into different modulations used in switched amplifiers. Section III analyzes limit-cycle amplifiers, and in Section IV an envelope amplifier using such principle is designed, measurements are taken from a prototype and compared against theoretical and simulated data. Section V concludes the paper.

## II. SWITCH-MODE MODULATIONS

Binary modulations were designed either to allow efficient amplification of signals, or to represent information in a bit-stream. Regarding efficiency, the main concern of this work, these modulations are used for switched-mode output stages. It is considered that the output has a stepped characteristic, either in current or voltage, such that the switching element dissipates minimum power in steady state (conducting current when the voltage across its terminals is zero, or not conducting any current when the voltage across its terminals is non-zero). In a general sense, practical implementations of such stages are binary and work in

voltage-mode, i.e, the waveform of the output voltage has a rectangular shape.

The main requirement of these modulations is that the modulated output includes the input signal spectra, such that the amplified signal can be obtained using the so called reconstruction filters, which should be passive and efficient in the sense that the only dissipative element should be the load itself.

Since continuous amplitude signals are being modulated by discrete amplitude ones, this class of modulators consist on an amplitude-to-time conversion, so that the information in the output signal is mostly modulated in time. This implies that the output amplitude resolution is dependent on the time resolution, which leads to the well-known quantization error, usually named quantization noise. On the other hand, if a continuous-time modulator is used instead of a discrete-time one, the output does not suffer quantization errors.

In the following subsections, a short overview regarding two widely used modulations, Pulse-Width Modulation (PWM) and Sigma-Delta ( $\Sigma\Delta$ ) Modulation will be given.

#### A. $\Sigma\Delta$ Modulation

$\Sigma\Delta$  (Sigma-Delta, or Delta-Sigma) modulation was developed to digitally encode analogue signals. However, since the output of a  $\Sigma\Delta$  modulator also includes the spectra of the input signal, regular reconstruction filters can be used to filter it out the amplified signal efficiently in the analogue domain.

A significant amount of theoretical work has been developed on  $\Sigma\Delta$  modulators namely in Analog-Digital conversion. The main interest behind this discrete-time modulation, comes from the fact that it shapes the quantization noise in the frequency domain. As so, it allows the trade-of between quantization-error and signal-bandwidth. A  $\Sigma\Delta$  modulator is represented in Fig. 1.

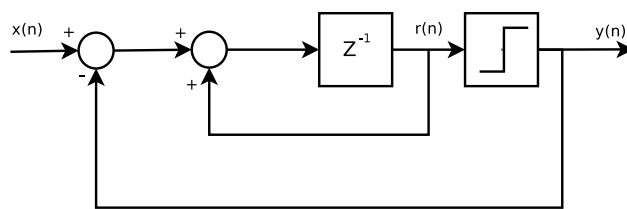


Fig. 1 -  $\Sigma\Delta$  Modulator

One method widely used to analyze this system, which can be seen in [13], is modeling the 1-bit quantizer output as a sum between the input signal and some uncorrelated noise  $e(n)$ . It has been experimentally observed that despite the fact that the behavior of the comparator by itself yields at the output distortion and not uncorrelated noise, the system's overall behavior matches the predictions made with this model.

From this assumption, the output will be split in two parts,  $y(n) = y_x(n) + y_e(n)$ , relative to the signal and error, respectively. From this, closed-loop gain for each of the input signals ( $x(n)$  and  $e(n)$ ) can be calculated in the  $Z$  domain, yielding

$$Y_x(z) = \frac{z^{-1}}{1 + \frac{z^{-1}}{1-z^{-1}}} Y(z) = z^{-1} Y(z) \quad (1)$$

$$Y_e(z) = \frac{1}{1 + \frac{z^{-1}}{1-z^{-1}}} = (1 - z^{-1}) E(z) \quad (2)$$

Using the transform  $Z \Rightarrow e^{2\pi j \frac{f}{f_s}}$ ,  $Y_e(z)$  yields

$$Y_e(f) = 2je^{-\pi j \frac{f}{f_s}} \sin\left(\pi \frac{f}{f_s}\right) E(f) \quad (3)$$

which means that the quantization noise is shaped like a sine in the frequency domain, having the zero at DC and the maximum at  $\frac{f_s}{2}$ .

To understand, in time domain, how can the quantization error be zero, let's consider that the  $\Sigma\Delta$  modulator only has two levels (0,1), that it is fed with a constant input  $x(n) = x_i$ , and that in  $t = 0$  the accumulator state is zero, then at each time-step  $k$ , the accumulated value in the integrator is equal to  $Nx_i - n_1$  where  $n_1$  is the number of '1' bits since the last time that the state of the accumulator has been zero, and  $N$  the number of total bits since then. One bit, is, of course, each rectangle pulse, of duration  $\frac{1}{T_{clock}}$ , since the  $\Sigma\Delta$  modulators are clocked.

From this, the period of the output can be calculated, and the periodic condition is

$$Nx_i - n_1 = 0$$

which is the case when the initial state is reached again. Simplifying this expression yields

$$x_i = \frac{n_1}{N}$$

It is possible to conclude from this equation that if  $x_i$  is a rational number the period would be the denominator of the reduced fraction, so that such fraction would be approximated with no error. Moreover, over time, an irrational number would be continuously approximated, in order to minimize the error. This yields for DC zero quantization error.

#### B. Pulse-Width Modulation

Pulse-Width Modulation (PWM) is constituted by a stream of width-modulated pulses, usually spaced in time by a constant period.

In Fig. 2 the system-level schematic of a PWM modulator can be seen. One of the main advantages of it, which widespread its use, is the fact that it is quite linear. Moreover, it is straightforward to use feedback if needed.

The amplitude-to-time conversion is made simply by comparing the input signal with a locally generated high-frequency triangular wave, which constitutes the dithering signal, a signal added to the input signal which purpose is to linearize the relay.

In [14] the spectral components of a discrete-time pulse-width modulated signal were calculated, using the Bessel series expansion, and the results are very close to cited simulation data, which makes it a good method to predict the

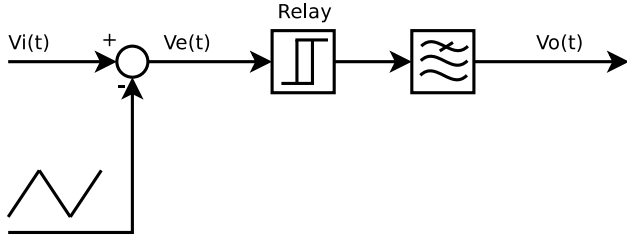


Fig. 2 - Simple open-loop Pulse-Width Modulator

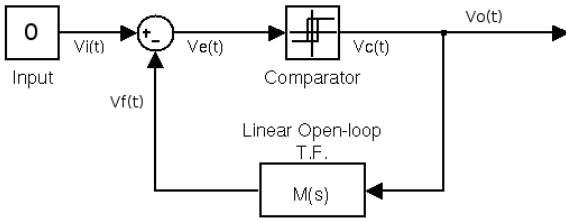


Fig. 3 - System architecture. The linear filter may include a propagation delay, or positive feedback.

effect of PWM modulation over frequency domain. Moreover, this modulation can also be used in continuous-time systems.

### C. Self-oscillating amplifiers

One drawback of the pulse-width modulator depicted in Fig. 2 is the need of a triangular-wave generator. Self-oscillating amplifiers, or limit-cycle amplifiers are a compact way of generating such PWM output through proper feedback, so that they oscillate and generate the proper dithering signal, thus not requiring any dedicated dithering generator.

However, in most of these systems frequency modulation also takes place, making its analysis more complex. The analysis of such systems, namely regarding to the oscillation condition, is done in [15] and with the Tsytkin's method in [16], however, none of these works considered the existence of a propagation delay in the loop, which have some implications on the methods cited. In the next section, the system-level circuit of limit-cycle amplifiers will be presented along with a detailed analysis taking such propagation delay into account.

## III. SYSTEM ANALYSIS

A zeroth-order limit-cycle amplifier [17] is depicted in Fig. 3. It is observed that for a stationary input the system is unstable and converges to a periodic rectangular wave. Such period is dependent on the input, and it is important to derive such dependency.

$M(s)$  is considered to be a linear time-invariant low-pass filter which also accounts for any propagation delay on the loop. The relay is considered to be ideal, without any propagation delay, and may or may not include hysteresis.

Assuming the output to be a square wave having period  $T$  and duty-cycle  $D$ , then the error signal  $V_e(t)$  at the input of

the relay can be calculated as

$$V_e(t) = V_i(t) - (2 \cdot D - 1) - \sum_{\substack{n=-\infty \\ n \neq 0}}^{+\infty} \frac{2 \sin(\pi n D)}{\pi n} \cdot e^{-j\pi n D} \cdot M(j2\pi \frac{n}{T}) \cdot e^{j2\pi \frac{n}{T} t} \quad (4)$$

where  $M(j\omega)$  stands for the linear filter transfer function.

It is possible to continue the analysis disregarding the value of  $V_i(t)$ , which is considered an independent variable, by considering that  $V_i(t)$  is a DC signal or a quasi-static signal compared to  $T$ , thus making  $V_i(0) = V_i(DT)$ . Considering that the hysteresis thresholds of the relay are  $+V_{th}$  and  $-V_{th}$  for the rising and falling edges respectively, then we conclude that  $V_e(0) = V_{th}$  and  $V_e(DT) = -V_{th}$  and by subtracting both equations (4), the following equation, which constitutes one necessary condition for system oscillation, must hold true:

$$\Psi(D, \omega) \equiv \sum_{n=1}^{+\infty} \frac{-4 \sin^2(\pi n D)}{\pi n} \cdot \text{Imag}[M(j\omega n)] = V_{th} \quad (5)$$

where  $\omega$  is the oscillating frequency, given by  $\omega = \frac{2\pi}{T}$ , and  $\text{Imag}(\cdot)$  is the imaginary-part operator. Additionally, four more necessary conditions must hold. Regarding the derivative of the  $V_e(t)$  signal, the following must apply

$$\begin{aligned} \frac{dV_e(t)}{dt} &> 0, \quad \forall t = kT, k \in \mathbb{Z} \\ \frac{dV_e(t)}{dt} &< 0, \quad \forall t = DT + kT, k \in \mathbb{Z}. \end{aligned} \quad (6)$$

These conditions are similar to those found in [15], and Tsytkin's method [16], however, these works do not considered a propagation delay, which is important when the delay of the switched power-stage is comparable to the oscillating period. And when such propagation delay is considered, in order to maintain causality from output to input, any change in the output of the relay can only affect the input, at least, after the propagation delay.

This means that the minimum pulse width is, at most, the propagation delay, and can be formally expressed making the positive pulse width  $T \cdot D$  and the negative pulse width  $T \cdot (1 - D)$  higher than  $\tau$ :

$$\omega < \frac{2\pi}{\tau} D \quad (7)$$

$$\omega < \frac{2\pi}{\tau} (1 - D). \quad (8)$$

When plotting the left-side of (5) over  $\omega$ , it is possible to solve the equation graphically, since the Y-axis represents the required  $V_{th}$  for the system to oscillate. As an example, Fig. 4 shows  $\Psi(D, \omega)$  for a second-order low-pass filter  $M(j\omega)$  without propagation delay. From the asymptotic behavior within the upper half-plane towards zero, we may conclude that the system does not fulfill (5) if no hysteresis is present on the system. Moreover, when a propagation delay is added to the linear filter, not only the equation has solutions for a relay without hysteresis, as it also has infinite

solutions for (5), which is depicted in Fig. 5. Regarding the multitude of solutions, the additional conditions 7 and (8) proposed in this work exclude all of them, except the one with the lowest-frequency. It is also verified that the propagation delay yields solutions for negative values of  $V_{th}$ .

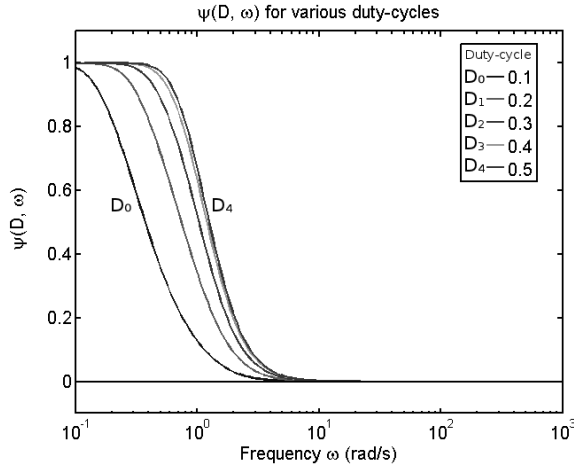


Fig. 4 - Equation 5 for a second-order low-pass filter without propagation delay.

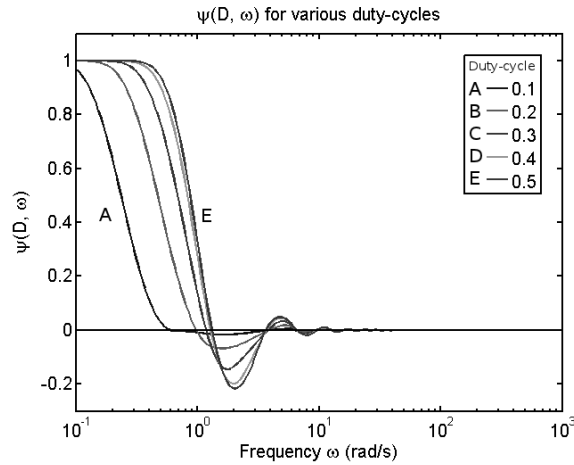


Fig. 5 -  $\Psi(D, \omega)$  for a second-order low-pass filter with propagation delay.

This suggests that it is possible to make the system oscillate at a higher frequency if a negative-hysteresis relay is used [18]. The input-output characteristic of such relay is depicted in Fig. 6.

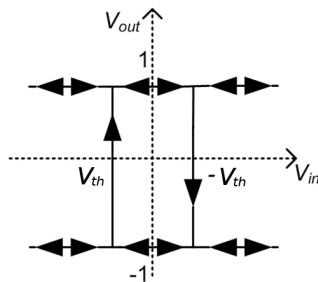


Fig. 6 - Input-output characteristic of a negative-hysteresis relay

The mathematical model was then used to predict oscillating frequency over the duty-cycle, and the results were

compared with system-level simulations for three different relay configurations. The linear filter  $M(j\omega)$  was considered to be a second-order low-pass filter with cut-off frequency  $\omega_c = 1(\text{rad/s})$  and a propagation delay  $\tau = 1(s)$ .  $V_{th}$  was considered to be  $-0.1V$ ,  $0V$  and  $0.1V$  respectively. The results are depicted in Fig. 7, and confirm the accuracy of the mathematical model.

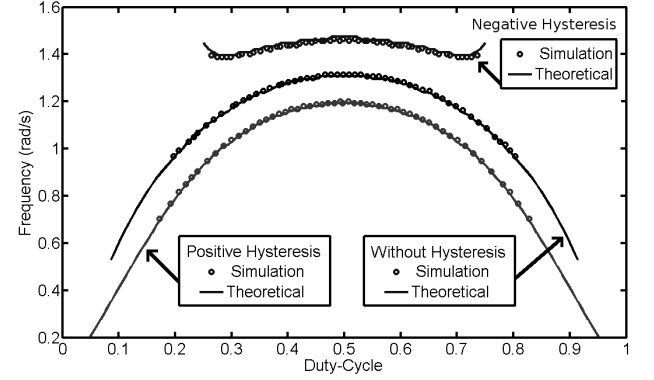


Fig. 7 - Comparison between the same system using different types of hysteresis. Theoretical data is overlapped, and shows to be an accurate prediction.

The last step was to derive the input-output characteristic, which is obtained after having the  $(D, \omega)$  solution-points for a particular system. Defining  $V_o \equiv (2D - 1)$ , and from (4) at  $t = 0$ , it is possible to derive

$$V_i(V_o) = V_o - V_{th} + \sum_{n=-\infty, n \neq 0}^{+\infty} \frac{2\sin(\pi n \frac{V_o+1}{2})}{\pi n} \cdot e^{-j\pi n \frac{V_o+1}{2}} \cdot M(j\omega n). \quad (9)$$

The results obtained by this expression were also compared with simulated data, and are depicted in Fig. 8.

This concludes the system analysis, leading to the design of a experimental prototype in the following section.

#### IV. EXPERIMENTAL RESULTS

To experimentally validate the mathematical model, a prototype for an RF envelope amplifier was designed. To design an amplifier with a less distorted characteristic than the one depicted in Fig. 8,  $\omega$  was made large compared with the cut-off frequency, which results in smaller  $M(j\omega n)$  terms in (9).

To select the order of the low-pass filter, Table I shows the stationary figures-of-merit. Keeping the same propagation delay  $\tau = 0.017s$  over all configurations, simulations were performed for three low-pass filters, having a cut-off frequency  $\omega_c = 1(\text{rad/s})$ . The self-oscillation frequency for 50% and 10% duty-cycles, the filter attenuation at those frequencies (which give a clue about the amplitude of the ripple) and the quasi-static linearity, represented by the Normalized Mean-Square Error (NMSE) of the  $V_o(V_i)$  error from the best-linear approximator taken from the Taylor series at  $V_o(0)$ .

In this table, it is clear that a larger phase margin in each filter's phase plot increases the frequency. First and second

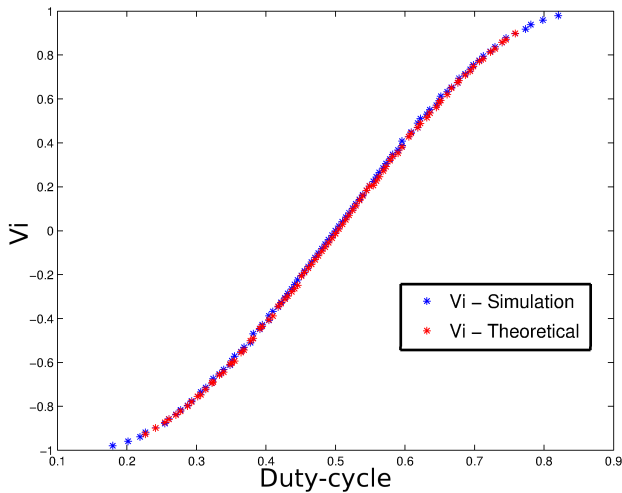


Fig. 8 -  $V_i(D)$  characteristic of a system using a relay without hysteresis, and a second-order low-pass filter plus propagation delay as  $M(j\omega)$ .

order filters, although having distinct oscillating frequency, have slightly the same attenuation at the fundamental frequency of the carrier. The third-order filter is clearly a worse option, because is much less linear, and the output ripple will be much larger.

Considering only the first and second-order filters, the first is slightly more linear, which would suggest to be the best option, however, the frequency in the first-order is approximately ten times higher than the second-order, and as so, it will have ten times more switching losses from  $\frac{1}{2}CfV^2$ . As so, a second-order filter was selected.

Regarding the output stage, since the output of an envelope amplifier is always positive, the quarter-bridge configuration [19] will be chosen in order to minimize propagation delay and simplify design, as the gate driver does not need any artificial delay to ensure break-before-make when controlling two series transistors. Moreover, the load impedance considered was approximately  $50\Omega$ , and the bandwidth was maximized taking into account the propagation delay inherent to the prototype, which was designed to be minimum, and that was kept under  $50ns$ . This led to an output filter whose cutoff frequency was 1.5MHz, and a final bandwidth of 180kHz.

The complete schematic can be found in Fig. 9, and a photograph of the prototype and the power converter unit used to provide the voltages is shown in Fig. 10.

The measurements regarding oscillating frequency and  $V_i(V_o)$  characteristic superimposed to theoretical and

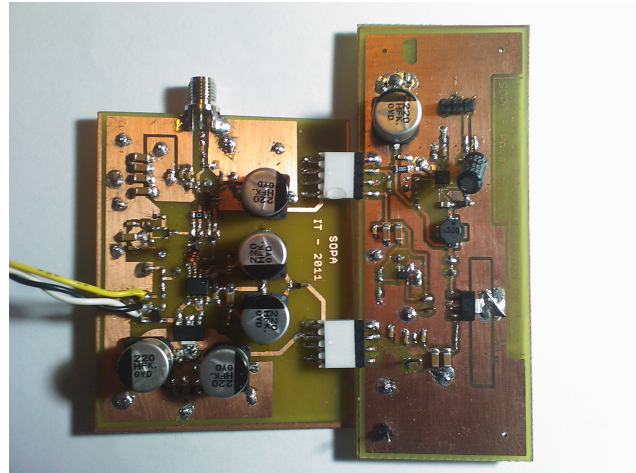


Fig. 10 - Photograph of the prototype. The right-hand side is the power supply and the left-hand side is the relay circuit.

SPICE simulated predictions are depicted in Fig. 11 and Fig. 12 respectively.

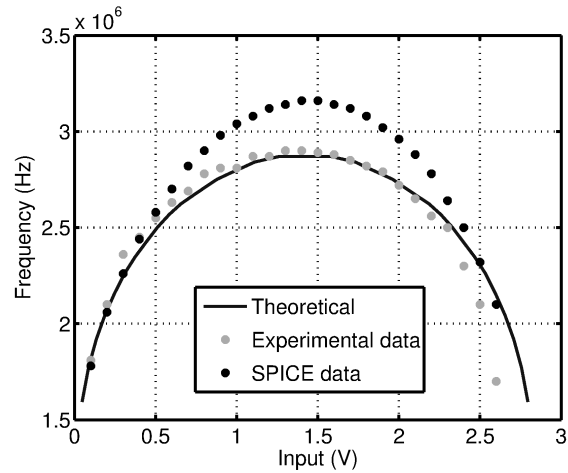


Fig. 11 - Self-oscillating frequency vs DC input amplitude, theoretical prediction, and SPICE simulation.

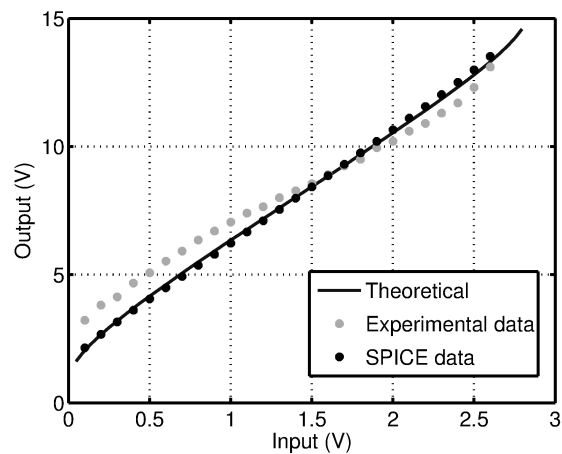


Fig. 12 -  $V_o(V_i)$  characteristic of the experimental prototype, theoretical prediction, and SPICE simulation.

The theoretical model fitted almost perfectly in the exper-

TABLE I

COMPARISSON BETWEEN LIMIT-CYCLE AMPLIFIERS, FOR DIFFERENT ORDER LOW-PASS FILTER.

| Filter order    | $\omega_{50\%}$ | $\omega_{10\%}$ | atten. $\omega_{50\%}$ | atten. $\omega_{10\%}$ | NMSE                 |
|-----------------|-----------------|-----------------|------------------------|------------------------|----------------------|
| 1 <sup>st</sup> | 94.55           | 33.42           | 39.0dB                 | 30.5dB                 | $1 \times 10^{-3}$   |
| 2 <sup>nd</sup> | 10.00           | 6.10            | 40.0dB                 | 31.6dB                 | $1.5 \times 10^{-3}$ |
| 3 <sup>rd</sup> | 1.67            | 1.25            | 17.4dB                 | 12.3dB                 | $6 \times 10^{-2}$   |

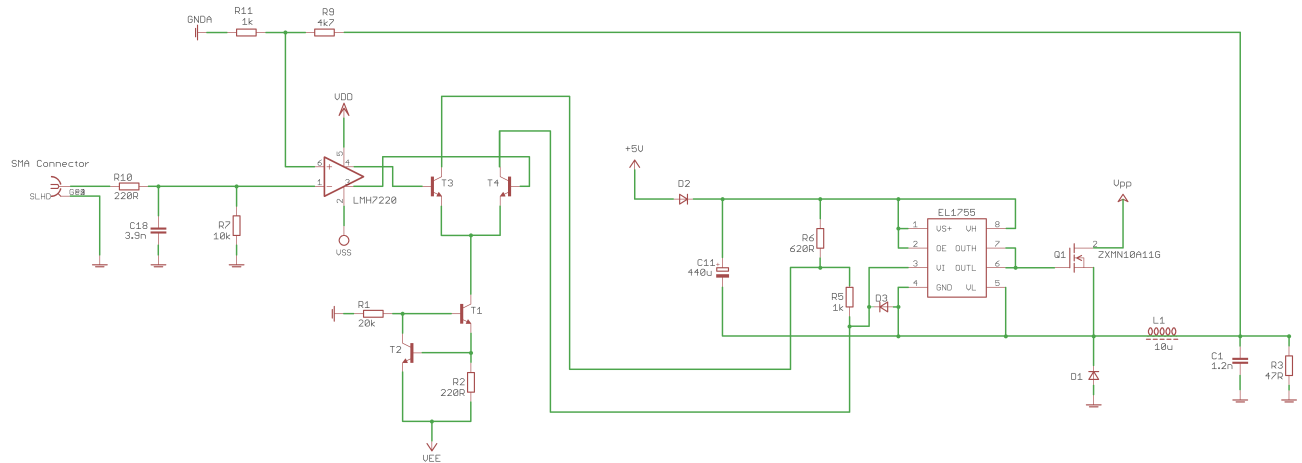


Fig. 9 - Schematic of the prototype.

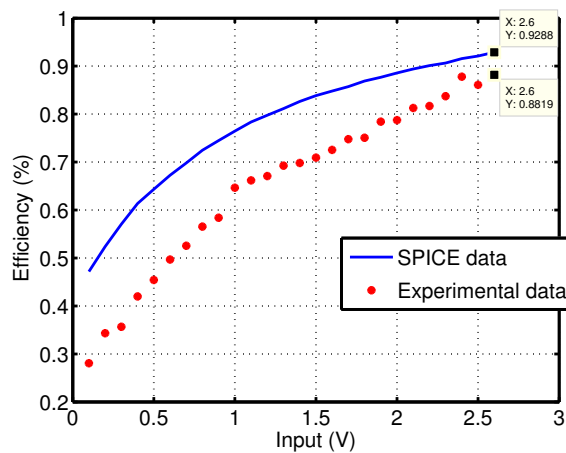


Fig. 13 - Amplifier DC efficiency.

imental results. The largest difference is that the Figure 11 is asymmetric, unlike the theoretical prediction. That is due to the fact that the power source has finite output resistance, hence, to yield a larger mean value the duty-cycle must be larger, leading to a lower frequency. Likewise, in Fig. 12, theoretical predictions fit SPICE and experimental data, and the latter exhibited an NMSE equal to 0.006 (-44.4370dB). Regarding efficiency results, depicted in Fig. 13, some differences exist between experimental and SPICE data. It is worth to mention that the SPICE model did not include the power converter designed to provide the different voltages required for different parts of the circuit, hence, the difference is caused by the power dissipated in the power converter, which has an 80% efficiency.

## V. CONCLUSIONS

In this work a lot of information was gathered in order to understand existing methods to modulate signals for an efficient amplification, in particular, for self-oscillating envelope amplifiers aimed at RF amplification.

This kind of amplifiers are very interesting, and do not require high-order low-pass filters in order to oscillate. The use of the inherent propagation delay of the circuit and hys-

teresis was exposed here, and a set of necessary conditions was derived which were used to predict the self-oscillation frequency as well the quasi-static transfer characteristic of a limit-cycle amplifier. Moreover, it was found that a negative hysteresis relay could increase the self-oscillating frequency, and that it only maintains a stable oscillation if a propagation delay exists in the loop. More detail on this topic can be found in [18].

Moreover, one self-oscillating power amplifier oscillating at  $2.8\text{MHz}$ , with a bandwidth of  $180\text{kHz}$  and capable of delivering up to  $15\text{W}$  was designed. The mathematical model have shown to be able to accurately predict the results of system-level simulations, as well the experimental data, both with an acceptable degree of accuracy.

Although the experimental prototype does not have enough bandwidth nor linearity to cope with modern high-frequency base-band signals, the main purpose of this design was to experimentally validate the mathematical model.

## VI. ACKNOWLEDGEMENT

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## REFERENCES

- [1] Jianhua He, P. Loskot, T. O'Farrell, V. Friderikos, S. Armour, and J. Thompson, "Energy efficient architectures and techniques for green radio access networks", in *Proc. 5th Int Communications and Networking in China (CHINACOM) ICST Conf*, 2010, pp. 1-6.
- [2] J. Gozalvez, "Green radio technologies [mobile radio]", *IEEE Veh. Technol. Mag.*, vol. 5, no. 1, pp. 9-14, 2010.
- [3] N. O. Sokal and A. D. Sokal, "Class-e, a new class of high-efficiency tuned single-ended switching power amplifiers", *IEEE J. Solid-State Circuits*, vol. 10, no. 3, pp. 168-176, 1975.
- [4] N. O. Sokal, "Class-e high-efficiency power amplifiers, from hf to

- microwave”, in *Proc. IEEE MTT-S Int. Microwave Symp. Digest*, 1998, vol. 2, pp. 1109–1112.
- [5] F. H. Raab, “Class-f power amplifiers with maximally flat waveforms”, *IEEE Transactions on Microwave Theory and Techniques*, vol. 45, no. 11, pp. 2007–2012, 1997.
  - [6] P. Cruz and N. B. Carvalho, “Papr evaluation in multi-mode sdr transceivers”, in *Proc. 38th European Microwave Conf. EuMC 2008*, 2008, pp. 1354–1357.
  - [7] Leonarde R. Kahn, “Single-sideband transmission by envelope elimination and restoration”, in *Proc. I.R.E.*, July 1952, vol. 40, pp. 803–806.
  - [8] P. F. Miaja, M. Rodriguez, A. Rodriguez, and J. Sebastian, “A linear assisted dc/dc converter for envelope tracking and envelope elimination and restoration applications”, in *Proc. IEEE Energy Conversion Congress and Exposition (ECCE)*, 2010, pp. 3825–3832.
  - [9] Feipeng Wang, *High Efficiency Linear Envelope Tracking and Envelope Elimination and Restoration Power Amplifier for WLAN OFDM Applications*, PhD thesis, University of California, San Diego, 2006.
  - [10] M. Vasić, O. García, J. A. Oliver, P. Alou, D. Diaz, J. A. Cobos, A. Gimeno, J. M. Pardo, C. Benavente, and F. J. Ortega, “High efficiency power amplifier based on envelope elimination and restoration technique”, in *Proc. IEEE Energy Conversion Congress and Exposition (ECCE)*, 2010, pp. 3833–3840.
  - [11] S.-A. El-Hamamsy, “Design of high-efficiency rf class-d power amplifier”, *IEEE Trans. Power Electron.*, vol. 9, no. 3, pp. 297–308, 1994.
  - [12] F. Arfaei Malekzadeh, R. Mahmoudi, and A. H. M. van Roermund, “A new approach for nonlinear metric estimation of limit cycle amplifiers”, in *Proc. European Microwave Conf. EuMC 2009*, 2009, pp. 1804–1807.
  - [13] Richard Schreier, Gabour C. Temes, and Steven R. Norsworthy, *Delta-Sigma Data Converters*, IEEE Press, 1996.
  - [14] Andreas C. Floros and John N. Mourjopoulos, “Analytic derivation of audio pwm signals and spectra”, *Audio Engineering Society*, vol. 46, no. 7/8, pp. 621–633, July/August 1998.
  - [15] E. Roza, “Analog-to-digital conversion via duty-cycle modulation”, *Transactions on Circuits and Systems II*, vol. 44, no. 11, pp. 907–914, 1997.
  - [16] S. Engelberg, *A Mathematical Introduction to Control Theory*, Imperial College Press, 2005.
  - [17] T. Piessens and M. S. J. Steyaert, “Behavioral analysis of self-oscillating class d line drivers”, *IEEE Trans. Circuits Syst. I*, vol. 52, no. 4, pp. 706–714, 2005.
  - [18] Manuel J. Duarte, Pedro M. Cabral, and José. C. Pedro, “Bandwidth improvement of a limit-cycle amplifier using a novel hysteretic relay”, in *Proceedings of The International Workshop on Integrated Nonlinear Microwave and Millimetre-wave Circuits*, 2012.
  - [19] Robert W. Erickson and Dragan Maksimović, *Fundamentals of Power Electronics, Second Edition*, Kluwer Academic Publishers, 2004.