A New Sequence of Numbers – Counting Regular Polytopes

Peter Cameron[1], Maria Elisa Fernandes[2], Dimitri Leemans[3]

1 – School of Mathematics and Statistics, University of St Andrews.
2 – CIDMA & Department of Mathematics, University of Aveiro.
3 – Département de Mathématique, Université Libre de Bruxelles.

In her recent research, Maria Elisa Fernandes and her collaborators have discovered a new sequence of integers, now listed in the *On-Line Encyclopedia of Integer Sequences* (OEIS) as A359367: 1, 1, 7, 9, 35, 48, 135, ...

This sequence emerges from the study of regular abstract polytopes whose automorphism groups is S_n. Using powerful computational algebra systems such as GAP and MAGMA, researchers can construct extensive atlases of algebraic structures, enabling the detection of patterns and the development of new mathematical insights. These tools play a central role in advancing mathematical knowledge by supporting both the formulation of new theories and the resolution of open problems.

Through extensive computational work, it was verified, up to $n = 16$, that:

- There is a unique polytope with group S_n and rank $r = n - 1$ $(n \geq 5)$.
- There is a unique polytope with group S_n and rank $r = n - 2$ $(n \geq 7)$.
- The number of polytopes with group S_n and rank $r = n - k$ remains constant for rank when $n \geq 2k + 3$.

These observations were rigorously proven for an arbitrary n and published in two articles in the journal *Advances in Mathematics*, the first in 2011 and the second in 2024. The sequence now cataloged as OEIS A359367 counts the number of regular abstract polytopes of a given rank with automorphism group S_n. The most recent term, corresponding to rank 11 and degree 17, was only recently computed. Unfortunately, algebraic computation has not allowed us to go beyond degree 17.

Finding a closed-form formula or recurrence for this sequence remains an open problem. Addressing this challenge requires optimizing existing algorithms and expanding the known terms of the sequence.

In recognition of her collaboration and research contributions, Maria Elisa Fernandes was awarded the Chaire Internationale IN of the Université libre de Bruxelles in both 2022 and 2025.

FIGURE 1
Regular Polytope- 120-cell.