Statistics of remote regions of networks

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We explore the statistical properties of regions within complex networks that are distant from vertices with high centralities, such as hubs or highly connected clusters. These remote regions play a pivotal role in shaping the asymptotic behaviors of various spreading processes and the features of associated spectra. Our study focuses on the probability distribution of the number of vertices located at a certain distance or beyond from a randomly chosen vertex in an undirected network. Previous work by Dorogovtsev, Mendes and Samukhin (2003) theoretically derived this distribution and its large-scale asymptotics for undirected uncorrelated networks.

We extend this analysis by employing numerical simulations and examining empirical data to explore a wide range of real undirected networks and their models, including both tree-like structures and networks with loops. We find that the inverse square law, which describes the probability distribution, remains valid even for networks with strong correlations. This law is observed in networks that exhibit the small-world effect and contain vertices with a degree of one, commonly referred to as leaves or dead ends.

We also identify specific classes of networks where the inverse square law does not hold. These include finite-dimensional networks and networks embedded in finite-dimensional spaces, where the structural constraints alter the expected distribution. Additionally, we observe that long chains of nodes in networks can reduce the range over which the inverse square law is applicable. Notably, we detect the presence of such long chains in the remote regions of the undirected projection of a large web domain, highlighting the complexity and variability in network structures.

Our comprehensive analysis underscores the importance of considering network topology when examining statistical properties and spreading processes, providing new insights into the behavior of complex networks far from central hubs.

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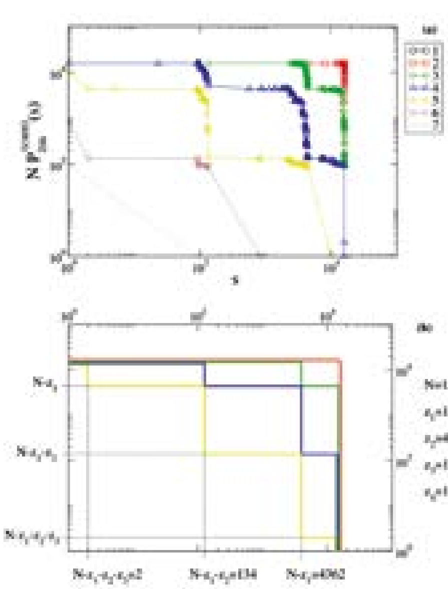

FIGURE 1
A network of hyperlinks between pages within Google's sites.

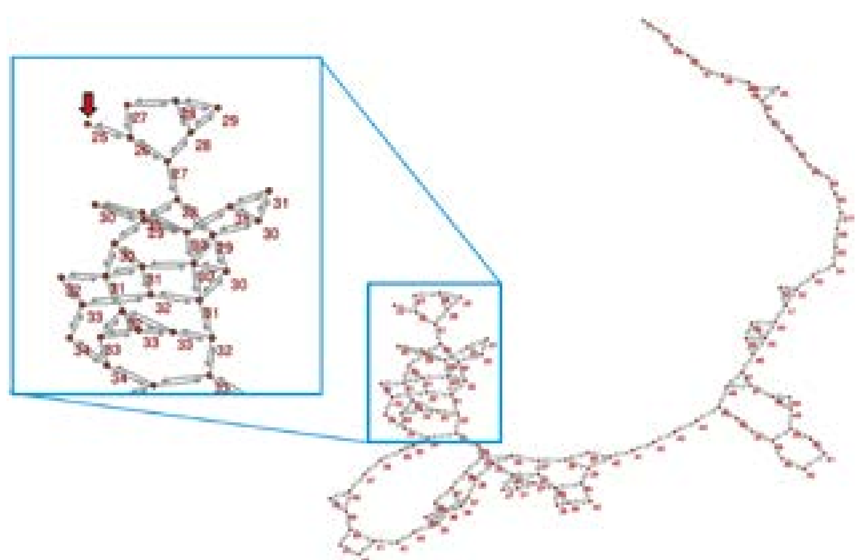

FIGURE 2
Stanford Web clusters formed by the vertices at a distance m = 25 or beyond from the largest hub in the undirected projection of the network, and edges between them.