An integral boundary fractional model to the world population growth

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Mathematical modeling is the process of describing a real world problem in mathematical terms, usually in the form of equations, and then using such equations both to help understand the original problem, and also to discover new features. Currently, applications and activities related to fractional (non-integer order) models have appeared in many fields of science, engineering and medicine, such as in HIV modeling and fluid dynamics. Fractional calculus has emerged as one of the most important interdisciplinary subjects in Mathematics, Biology, and Engineering. It has been shown that differential equation models involving fractional derivatives describe certain phenomena better than traditional integer-order differential equation models.

Many analytical and numerical methods have been proposed in the literature to describe the world population growth. In this work, we formulate a new mathematical model that consists of a fractional (non-integer order) differential equation of order between two and three, involving a psi-Caputo fractional derivative subject to initial conditions on the unknown function and its first-order derivative as well as an integral boundary condition that depends on the unknown function. We find a non-integer order alpha and a simple function psi for which the solution of our fractional model describes given real data better than available models.

First we proved the existence and uniqueness of solution to the proposed model. Our proof is constructive and an explicit formula for the solution is given, which depends on the fractional order alpha and the arbitrary smooth function psi. By using real data, we have proved that describing the population growth by a psi-fractional differential equation subject to an integral boundary condition is more accurate than all the previous classical and fractional models available in the literature.

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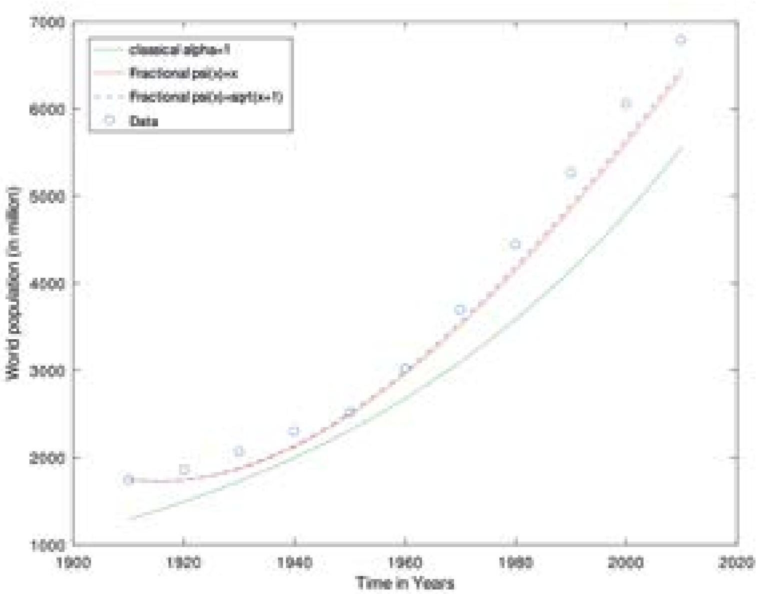

FIGURE 1
Real data versus classical integer-order model (particular case of our model with alpha=1) versus standard fractional model (particular case of our model with psi(x)=x) versus the new proposed psi-fractional model with psi(x) = sqrt(x+1). Figure taken from https://doi.org/10.1016/j.chaos.2023.113151.