The spectrum of the *H*-join of arbitrary graphs – the walk matrix approach

Domingos M Cardoso[1], Sofia J Pinheiro [1], Helena Gomes[2]

The results obtained in [D.M. Cardoso, H. Gomes, S.J. Pinheiro, The *H*-join of arbitrary families of graphs – the universal adjacency spectrum, Linear Algebra Appl. 648 (2022): 160-180] resolve the nearly five decades problem raised by a remark in [A. J. Schwenk, Computing the characteristic polynomial of a graph, Graphs and Combinatorics (Lecture notes in Mathematics 406, eds. R. Bary and F. Harary), Springer-Verlag, Berlin, (1974): 153-172]. Indeed, since 1974 the expressions for the determination of the characteristic polynomial as well as the entire spectrum of the adjacency matrix of the *H*-join of a family of graphs $G = \{G_1, \ldots, G_p\}$ also called generalized composition $H\{G_1, \ldots, G_p\}$, where *H* is an arbitrary graph with *p* vertices (see an example in Figure), in terms of the characteristic polynomial (respectively the spectra) of adjacency matrices of its components, that is, graphs in *G*, and an associated matrix, were limited to families *G* of regular graphs. In his 1974 article, Schwenk wrote the following remark: in general, it does not appear likely that the characteristic polynomial of the adjacency matrix of the generalized composition $H\{G_1, \ldots, G_p\}$ can always be expressed in terms of the characteristic polynomials of adjacency matrices of the graphs $G_1, \ldots, G_p$. In the 2022 above article, considering an arbitrary graph *H* with *p* vertices and a family of arbitrary graphs $G = \{G_1, \ldots, G_p\}$, based on a walk matrix approach, the authors obtained an expression for the determination, in an effective way, of the characteristic polynomial as well as the entire spectrum of the universal adjacency matrix of the generalized composition $H\{G_1, \ldots, G_p\}$, in terms of the characteristic polynomials (respectively the spectra) of the universal adjacency matrices of the components and an associated matrix. Note that the universal adjacency matrix of a graph *G* is $U(G) = \alpha A(G) + \beta I + \Upsilon J + \delta D(G)$, where $\alpha \neq 0$ and when $\alpha = 1$ and $\beta = \Upsilon = \delta = 0$, $U(G)$ coincides with the adjacency matrix $A(G)$.

1 – Department of Mathematics & CIDMA, University of Aveiro.
2 – Escola Superior de Educacao & CIDMA, Instituto Politecnico de Viseu.

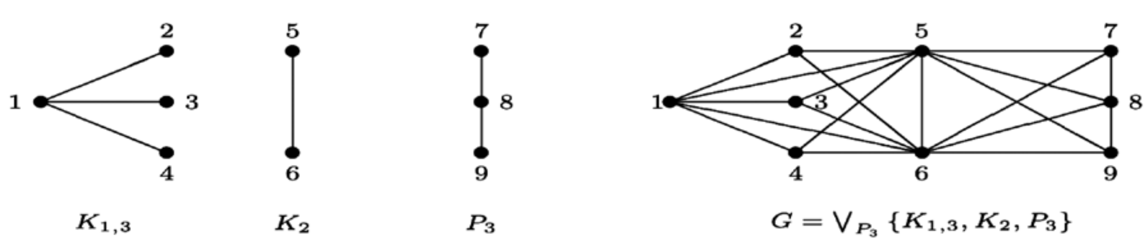

FIGURE 1
$P_3\,[K_{1,3}, K_2, P_3]$