High-root topological insulators

Anselmo Marques[1], Luísa Madail[1,2], Ricardo Dias[1]

1 – Department of Physics & i3N, University of Aveiro
2 – International Iberian Nanotechnology Laboratory, Braga

Topological insulators (TIs) are materials with an electronic insulating bulk and conductive edge states, which are protected by the presence of certain symmetries in the model, meaning that they are robust against perturbations that respect these symmetries. Recently, it has been shown that to obtain the topological invariants of a class of TIs with finite energy topologically protected states, modified approaches are required such as the squaring of the Hamiltonian. This approach defines the so-called squared-root topological insulators (SRTIs) and relies on the fact that the square of the SRTI Hamiltonian in the Wannier basis is a block diagonal matrix. More precisely, it is the direct sum of two blocks that have the same spectrum but different eigenstates (one of the blocks being the Hamiltonian of a known topological insulator).

An extension of the SRTI concept has been proposed in our works below, which may be described as a generalized root of order 2n (a square-root operation applied n times) of a tight-binding model. The latter model may be topological to start with or, if not, topological behavior may be introduced at any given step of the sequence of n square-root operations. We label these Hamiltonians as High-Root Topological Insulators (HRTIs). Squaring n times a HRTI, we obtain an outwards directed rooted tree (arborescence in graph theory) which connects the HRTI to the multiple diagonal blocks in the n-times squared Hamiltonian [with at least one of the arborescence nodes/blocks corresponding to the Hamiltonian of the known topological insulator].

In Fig. 1, we show an example of a 23-root 1D linear chain on top, which requires 3 squaring operations to arrive at the smaller parent model on the bottom, with the respective energy spectrum at each step shown at the right. In Fig. 2, starting from the 2D Haldane model in (c), we derive its square- and quartic-root versions in (b) and (a), respectively. The bottom row shows their respective energy spectra, where the number of topological edge bands is seen to increase with the root degree of the model.

FIGURE 1
Three times squarable sine-cosine topological chain on top, with its successive squared version below, whose energy spectra are shown in the same color at the right.

FIGURE 2
Plaquette of the Haldane model in its (a) quartic-root, (b) square-root, and (c) original version. The respective energy spectrum of each case for a ribbon geometry (open along a_1 and periodic along a_2) is shown below in (d)-(f), with bulk (edge) bands in blue (red).