Newton's problem of minimal resistances

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A rigid body moves in a highly rarefied medium. One can think, for instance, of an artificial satellite of the Earth moving at a low altitude (100 to 1000 km), where the atmosphere is extremely thin. As a result of collisions of the body with the medium particles, the force of resistance is created, which acts on the body and slows down its velocity. The problem is to find the convex body with fixed length and maximal width that has the smallest resistance.

This problem was studied by I. Newton (1687) in the class of rotationally symmetric bodies, and is now one of classical problems that gave rise to Calculus of Variations. Mathematically, the problem amounts to minimizing the integral $\int_0^1 \frac{1}{1+\varphi'(x)^2}\,dx$ in the class of concave functions φ such that $0 \le \phi \le M$, where M is the parameter of the problem. In Fig. 1, Newton's solution for $M=2$ is shown.

The new life to the problem was given in the 1990s, when it became clear that in the general case (without any assumption of symmetry) the optimal shape does not coincide with Newton's one. Mathematically, the more general problem is to minimize $\iint_\Omega \frac{1}{1+\nabla u(x,y)^2}\,dxdy$ in the class of concave functions $0 \le u \le M$ on the unit disk Ω. The problem remains open until now. The results of numerical simulation on the optimal shape are shown in Fig. 2, when M equals 0.4 (a), 0.7 (b), 1 (c), and 1.5 (d). Both numerical simulation and some theoretical arguments lead to the conjecture, first stated in 1990s, that the regular part of the surface of the optimal body can be foliated by line segments (similarly to cylindrical and conical surfaces). This conjecture is proved in the papers [1,2] by the author. The proof is based on the novel method of bilateral variation of a convex body called *nose stretching*.

FIGURE 1

A. Plakhov. Method of nose stretching in Newton's problem of minimal resistance. Nonlinearity 34, 4716-4743 (2021).

FIGURE 2

A. Plakhov. A solution to Newton's least resistance problem is uniquely defined by its singular set. arXiv:2109.14207v3 (2021).