CGO-Faddeev approach for complex conductivities with regular jumps in two dimensions

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One of the fundamental problems in the area of Inverse Problems is the so-called Calderón problem, which asks if it is possible to reconstruct uniquely the conductivity of a body from boundary measurements. In the last decades, the need for non-invasive imaging has been key in diverse areas such has geological prospecting, civil engineering and medical imaging and this problem is the basis of many such modern methods.

Unfortunately, besides the well-known case of A. Astala and L. Paivarinta (Annals of Mathematics, 2006) who solve the problem for non-physical situation of zero frequency, mathematicians were not able to overcome the requirement of the conductivity being at least one-time differentiable in solving this problem. This condition is never fulfilled in practical applications where changes between tissues and materials are common. In this work a novel approach was proposed which is based on a new original concept of admissible points which allows to study conductivities with jumps. This work combines the problem with transmission problems and achieves a reconstruction result for positive frequencies. It is based on the introduction of a new set of complex geometrics optic solutions, essential in the literature for the proofs of reconstruction and uniqueness, and on the adaptation of the scattering data. In this work indications were given that the problem is solvable if the proposed line of research is developed.

This work is part of the master thesis of Ivan Pombo and has been so well received by the community that it is now published as a paper in the journal „Inverse Problems", the principal journal in the field with Ivan Pombo as the sole author.

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FIGURE 1
One can see the potential medical applications of reconstructing the conductivity through a measuring system consisting of electrodes around the body.

FIGURE 2
Application of diverse linear independent current patterns and measure of the corresponding voltages, through a set of electrodes, allows the reconstruction of a saline and agar phantom simulating the lungs and heart.