# Mapping the structure of directed networks: Beyond the bow-tie diagram

Gábor Timár[1,2], Alexander V. Goltsev[1,2], Sergey N. Dorogovtsev[1], José F. F. Mendes[1]

The majority of large directed networks (such as the WWW and Twitter) have a central core – the giant strongly connected component (GSCC) – that consists of nodes that can be reached by any other node in the core via a directed path. The nodes that can be reached from the GSCC constitute the giant out-component (GOUT) and nodes that can reach the GSCC make up the giant in-component (GIN). This is the well-known "bow-tie" picture of directed networks.

The above representation gives a very informative view of the structure of directed networks, however a large fraction of nodes in the given network is left uncategorized. We propose a scheme by which all nodes in an arbitrary directed network are categorized: a hierarchical multilayer structure of "tendril layers" connected to the giant components (Fig. 1a). We propose an efficient computational algorithm to find all these new components, thereby giving a complete characterization of the structure of an arbitrary directed network. The newly introduced components permit a much richer organization of nodes than previously noted (Fig. 1b).

We also introduce a generalized connectivity function and the related susceptibility that may be used to estimate the distance from the percolation transition in directed networks: the point where the giant components disintegrate into small directed components, or vice versa, where small components merge to form the GSCC, GIN and GOUT. We indicate that near this transition point the main contribution to the susceptibility is from the newly-categorized nodes in the tendril layers attached to the giant components.

1 — Department of Physics & I3N, University of Aveiro
2 — A. F. Ioffe Physico-Technical Institute, Russia

**FIGURE 1**

a) Schematic view of the complete structure of an arbitrary directed network.
b) The extension of the "bow-tie" diagram, including a hierarchy of tendril components attached to the giant components in a directed network.