Highest rank of a polytope for An

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An Aveiro Theorem

A polytope is a higher-dimensional generalization of a polygon in 2 dimensions or a polyhedron in 3 dimensions. Regular polytopes, those with highest degree of symmetry, can be completely described by their group of symmetries, generated by a set of reflections (the size of this set is the rank of the polytope corresponding, in nice geometric cases, to the dimension of the space in which it is embedded). An interesting question is: Given a finite group G, describe all the regular polytopes with group of symmetries G; or, at least, describe the largest possible rank of such a polytope. It is not hard to show that the largest possible rank for the symmetric group Sn is n–1. The challenge was to consider the alternating group An instead. In 2012 Dimitri Leemans, M. Elisa Fernandes and Mark Mixer made the following conjecture:

The maximal rank of an abstract regular polytope, whose group of symmetries is an alternating group An is [(n–1)/2] for n>11 (The group A11, exceptionally, is the group of a regular polytope of rank 6).

They worked during the following years trying to find a solution to this problem and firstly they constructed abstract regular polytopes for the alternating group having the hypothetic highest possible rank. This supported their belief on the conjecture made in 2012. After some years of work on this problem, with a lot of travelling between New Zealand, Belgium, the United States and Portugal, they worked with Peter Cameron, a well known Australian mathematician (https://pt.wikipedia.org/wiki/Peter_Cameron) who felt attracted by the study of abstract regular polytopes and decided to help them solving this conjecture. In September 2015, while Peter Cameron and Dimitri Leemans were visiting Maria Elisa Fernandes, a researcher from CIDMA, they were able to finish the 42 pages proof, that can now be found in the *Proceedings of London Mathematical Society* (https://doi.org/10.1112/plms.12039). Peter Cameron refers to this result as one of the deepest results of his career. In his blog he calls it "An Aveiro Theorem" (https://cameroncounts.wordpress.com/2015/10/18/an-aveiro-theorem/).

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