

# Approximation in Morrey spaces

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Morrey spaces are widely used in applications to regularity properties of solutions to partial differential equations (PDE), including the study of Navier–Stokes equations. They are usually attributed to C.B. Morrey due to the celebrated paper On the solutions of quasi-linear elliptic partial differential equations, Trans. Amer. Math. Soc. 43 (1938), 126–166.

Although Morrey spaces are not separable in general, they allow to describe local properties of functions better than Lebesgue spaces. This feature is particularly important in applications. Nowadays there are many papers in the literature dealing with this kind of spaces and their generalizations.

Our work gives a contribution to the problem of the approximation by smooth functions with compact support in Morrey setting. Considering a number of known

and new vanishing properties for Morrey functions, we have introduced a new subspace of Morrey spaces whose elements can be approximated by infinitely differentiable compactly supported functions. As a consequence, we have obtained an explicit description of the closure of the set of such functions in Morrey norm.

This research was motivated from the lack of density of smooth functions in Morrey spaces. We believe that our contribution will be of strong interest either for the theory of function spaces itself or from the applications point of view. On the other hand, our results are expected to be very useful in Harmonic Analysis (including Calderón-Zygmund theory) in Morrey spaces. Moreover, we are convinced this will be highlighted in new papers stimulated by this research.