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Think, Solve, Win: The Open Middle Game

Pensa, Resolve, Ganha: O Jogo Open Middle

Piensa, Resuelve, Gana: El Juego Open Middle

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Abstract

This paper introduces an educational game designed to foster critical thinking, problem-solving, and collaborative learning through mathematics. Inspired by the Open Middle Math Problems Framework, the game offers an engaging way to challenge students' mathematical thinking by moving beyond routine exercises into a collaborative and competitive game environment. Players, organized in teams, engage with a variety of mathematical challenges that demand strategic thinking, precise execution, and time management.
Open Middle Problems are characterized by having fixed start and end points – all students start with the same initial problem and there is typically one correct or optimal answer. However, they allow for diverse solution strategies and approaches along the way. This "open middle" encourages students to explore multiple pathways, develop mathematical reasoning, and communicate their thought processes, making it a powerful tool for cultivating a deeper understanding of mathematical concepts.
Beyond enhancing student engagement in mathematics, the game is a valuable resource for educators seeking to make mathematics instruction more interactive and impactful. By transforming traditional problem-solving tasks into playful competitions, the game bridges the gap between academic rigor and entertainment. It encourages students to persist through complex problems, reflect on their strategies, and develop confidence in their mathematical abilities.

Keywords: educational games; mathematics; open middle problems; problem-solving.

Resumo

Este artigo introduz um jogo educativo concebido para promover o pensamento crítico, a resolução de problemas e a aprendizagem colaborativa através da matemática. Inspirado na

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estrutura Open Middle Math Problems, o jogo oferece uma forma envolvente de desafiar o pensamento matemático dos alunos, indo além dos exercícios de rotina para um ambiente de jogo colaborativo e competitivo. Os jogadores, organizados em equipas, envolvem-se com uma variedade de desafios matemáticos que exigem pensamento estratégico, execução precisa e gestão do tempo.
Os problemas matemáticos Open Middle caracterizam-se por terem pontos de partida e de chegada fixos – todos os alunos começam com o mesmo problema inicial e normalmente existe uma resposta correta ou ótima. No entanto, permitem diversas estratégias e abordagens de resolução ao longo do caminho. Este "meio aberto" incentiva os alunos a explorar múltiplos caminhos, a desenvolver o raciocínio matemático e a comunicar os seus processos de pensamento, tornando-o uma ferramenta poderosa para cultivar uma compreensão mais profunda dos conceitos matemáticos.
Para além de aumentar o envolvimento dos alunos na matemática, o jogo é um recurso valioso para os educadores que procuram tornar o ensino da matemática mais interativo e impactante. Ao transformar as tarefas tradicionais de resolução de problemas em competições lúdicas, o jogo estabelece a ponte entre o rigor académico e o entretenimento. Incentiva os alunos a persistir em problemas complexos, a refletir sobre as suas estratégias e a desenvolver confiança nas suas capacidades matemáticas.

Palavras-chave: jogos educativos; matemática; problemas open middle; resolução de problemas.

Resumen

Este artículo introduce un juego educativo diseñado para promover el pensamiento crítico, la resolución de problemas y el aprendizaje colaborativo a través de las matemáticas. Inspirado en la estructura de los Open Middle Math Problems, el juego ofrece una forma atractiva de desafiar el pensamiento matemático de los estudiantes, yendo más allá de los ejercicios rutinarios para crear un entorno de juego colaborativo y competitivo. Los jugadores, organizados en equipos, se enfrentan a una variedad de desafíos matemáticos que requieren pensamiento estratégico, ejecución precisa y gestión del tiempo.
Los problemas matemáticos Open Middle se caracterizan por tener puntos de partida y de llegada fijos: todos los estudiantes comienzan con el mismo problema inicial y, por lo general, hay una única respuesta correcta u óptima. Sin embargo, estos problemas permiten diversas estrategias y enfoques de resolución a lo largo del proceso. Este "medio abierto" anima a los estudiantes a explorar múltiples caminos, desarrollar el razonamiento matemático y comunicar sus procesos de pensamiento, convirtiéndose en una herramienta poderosa para fomentar una comprensión más profunda de los conceptos matemáticos.
Además de aumentar la participación de los estudiantes en el aprendizaje de las matemáticas, el juego es un recurso valioso para los educadores que buscan hacer la enseñanza más interactiva y significativa. Al transformar las tareas tradicionales de resolución de problemas en competencias lúdicas, el juego crea un puente entre el rigor académico y el entretenimiento. Motiva a los estudiantes a perseverar en problemas complejos, reflexionar sobre sus estrategias y desarrollar confianza en sus habilidades matemáticas.

Palabras clave: juegos educativos; matemáticas; problemas de tipo Open Middle; resolución de problemas.

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Introduction

Using games in the classroom provides several benefits that help transform the learning experience. Games minimize the traditional fear associated with mathematics, making it more engaging and accessible for students (Boaler, 2022), and promote active participation by encouraging students to interact dynamically with mathematical concepts (Kapp, 2012). They also stimulate peer interaction and collaborative problem-solving, fostering curiosity and attentive discussion (Chapin et al., 2009). Beyond engagement, games help automate reasoning processes and support the development of problem-solving strategies, while enabling self-assessment for reflection and growth (Wiliam, 2011).

Theoretical Framework

Open Middle Math problems (Kaplinsky, 2020) represent a pedagogical innovation in mathematics education, offering a potent means of instruction by strategically integrating structural frameworks with opportunities for creative mathematical exploration. A defining characteristic of these problems is their clear specification of initial conditions and desired outcomes, while intentionally leaving the intermediate solution pathways open to diverse and varied approaches. This contrasts markedly with traditional mathematical exercises that often prioritize the algorithmic application of a predetermined sequence of procedural steps. Instead, these problems actively cultivate students' capacities for logical deduction, critical analysis, and innovative mathematical reasoning.

A relevant advantage of employing Open Middle Math problems lies in their inherent capacity to accommodate students at their current level of mathematical understanding while simultaneously providing a platform for deeper conceptual engagement and the cultivation of advanced mathematical thinking (Kaplinsky, 2020). Consider, for instance, the classic magic square problem – arranging the integers 1 through 9 within a 3x3 grid so that the arithmetic sum of each row, column, and principal diagonal is equivalent. While this problem is well-known beyond the Open Middle framework, it exemplifies the characteristics of such tasks, possessing a clearly defined beginning and end but allowing for multiple pathways and strategies to reach the solution. It immediately necessitates the identification of underlying numerical patterns, iterative experimentation with various organizational strategies, and subsequent refinement of these approaches based on emergent insights (Boaler, 2022). Problem types of this nature compel students to move beyond rote memorization, instead requiring critical analysis of fundamental mathematical principles and iterative testing until a viable solution is achieved.

Furthermore, the incorporation of Open Middle Math problems into the curriculum serves as a catalyst in the development of a "growth mindset" among learners. The inherent ambiguity and openness of the intermediate problem-solving stages foster an environment wherein initial unsuccessful attempts are framed not as failures, but rather as integral and informative components of the learning trajectory. This pedagogical approach normalizes the experience of intellectual struggle as a natural and productive aspect of mathematical inquiry, ultimately enhancing the depth

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and durability of learning outcomes (Boaler, 2022). By deliberately mitigating the affective barrier associated with the fear of immediate incorrect responses and instead emphasizing the cognitive processes inherent in mathematical investigation, Open Middle Math problems effectively cultivate intellectual perseverance and promote sustained engagement with challenging mathematical tasks.

One of the most remarkable pedagogical benefits derived from the implementation of Open Middle Math problems is their transformative influence on the nature and quality of students' mathematical discourse. When actively engaged in addressing these problem types, students naturally gravitate towards the utilization of more precise and sophisticated mathematical terminology, formulate testable conjectures regarding the inherent relationships between mathematical entities, and articulate the logical justifications underpinning their proposed solution strategies. This rich mathematical dialogue is in direct alignment with established principles of best practice in mathematics education as recognized by leading international professional organizations (National Council of Teachers of Mathematics, 2018). The inherent duality of these problems – their structured framework coupled with their open-middle nature – provides ample and meaningful opportunities for students to critically evaluate the mathematical reasoning of their peers and to meticulously articulate the procedural and conceptual underpinnings of their own mathematical processes.

Literature Review

Teachers who regularly integrate Open Middle Math problems into their instructional classes frequently observe a marked increase in student engagement and a concomitant enhancement in collaborative interaction among students. The potential for the emergence of multiple valid solution pathways empowers students to develop a greater sense of intellectual agency and to feel more confident in proposing and defending their unique mathematical ideas. Consequently, the classroom environment evolves into a dynamic and interactive space characterized by the open exchange of mathematical insights, the comparative analysis of diverse solution methodologies, and the explicit articulation of individual mathematical reasoning – a pedagogical paradigm strongly supported by empirical research in mathematics education (Chapin et al., 2009).

When students engage in substantive discussions about their mathematical reasoning with peers, they refine their capacity for self-assessment and become more receptive to alternative and potentially more efficient problem-solving methodologies (Chapin et al., 2009). These practices are further supported by research on the cognitive benefits of retrieval and reconsolidation in learning, emphasizing the value of revisiting and restructuring knowledge collaboratively (Bjork & Bjork, 2019). This fosters a classroom culture that values intellectual risk-taking, collaborative inquiry, and sustained effort in the face of mathematical challenges (Boaler, 2022). Moreover, game-based and hands-on strategies have been shown to strengthen engagement and problem-solving persistence, particularly when students explore solution paths creatively (Hall et al., 2024). Over time, these cultivated cognitive and collaborative skills prepare students to engage with increasingly sophisticated mathematical content and to become more confident and adept problem-solvers (Hiebert & Grouws, 2007).

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Moreover, Open Middle Math problems serve as invaluable diagnostic instruments for educators seeking to identify specific areas of conceptual misunderstanding among their students. Given the multiplicity of potential solution trajectories inherent in these problems, the approach adopted by a student can provide significant and nuanced insights into their underlying mathematical comprehension, revealing both areas of strength and areas requiring further pedagogical attention. In this regard, these problem types offer teachers a far more granular and informative assessment of student thinking compared to the only evaluation of final numerical answers, thereby facilitating the implementation of more targeted and efficacious instructional interventions (Wiliam, 2011).

Furthermore, Open Middle Math problems exhibit a remarkable degree of adaptability and can be effectively deployed across a wide spectrum of academic grade levels and mathematical content domains (Kaplinsky, 2020). In the elementary grades, these problems might focus on the development of foundational number sense and counting principles. In middle school, they could be strategically employed to explore fundamental algebraic concepts and the identification of mathematical patterns. And in secondary or even tertiary educational settings, they can be adapted to address more advanced and conceptually complex topics such as optimization techniques or fundamental principles of calculus. This inherent adaptability allows educators to precisely calibrate the cognitive demand and mathematical complexity of the problems to effectively address the specific learning needs of their diverse student populations.

The cognitive engagement characterized by "productive struggle" that students experience when grappling with Open Middle Math problems contributes significantly to enhanced neural processing and the more robust and enduring retention of mathematical knowledge (Bjork & Bjork, 2019). Additionally, the iterative process of exploring and evaluating multiple potential solution strategies before converging on an optimal approach closely mirrors the deliberate practice routines employed by experts across various professional domains in their pursuit of mastery.

Upon the successful completion of Open Middle Math problems, the strategic deployment of thoughtfully crafted reflective prompts can further enhance student learning. These prompts encourage students to engage in metacognitive reflection on the core mathematical concepts that were central to the problem, the diverse solution strategies that were attempted (both successful and unsuccessful), and the underlying mathematical connections that were revealed through the problem-solving process. Questions such as "What were your initial mathematical observations?" or "What criteria did you employ to determine the relative efficacy of different solution pathways?" actively engage students in a critical analysis of their own problem-solving heuristics. This type of metacognitive reflection not only deepens their immediate mathematical understanding but also cultivates transferable problem-solving skills that are applicable across a wide range of academic and real-world contexts.

This pedagogical orientation is strongly supported by established empirical research on effective mathematics instruction (Hiebert & Grouws, 2007). By actively engaging in reflective thinking about their mathematical work, students explore fundamental mathematical ideas in a more meaningful and conceptually integrated manner. They begin to recognize the intrinsic interconnectedness of seemingly disparate mathematical concepts and develop a flexible repertoire of strategies for approaching novel and complex problems from multiple conceptual vantage points.

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The broader impact of consistently integrating Open Middle Math problems into the mathematics curriculum extends beyond their immediate application within classroom activities. As students become increasingly accustomed to this pedagogical approach, they cultivate robust mathematical thinking habits that transfer to other academic disciplines and persist throughout their educational trajectories (Kaplinsky, 2020). These problem types fundamentally reshape students' epistemological stance towards mathematics, transforming them from passive recipients of prescribed rules into active and engaged mathematical thinkers and sense-makers (Boaler, 2022).

In summation, Open Middle Math problems represent more than a mere instructional technique; they offer a transformative pedagogical framework for cultivating a mathematics classroom environment that actively fosters exploration, mathematical discovery, and deep conceptual understanding. By thoughtfully and consistently employing this approach, educators empower students to perceive mathematics not as a static and immutable body of rules, but rather as a dynamic, flexible, and inherently creative domain of inquiry where intellectual curiosity and unwavering perseverance are recognized as essential attributes of successful mathematical thinkers.

Goals and Methodology

Two research questions guide this study:

1. *How can Open Middle Problems be incorporated into a game-based learning (GBL) methodology?*
2. *How do students perceive the use of educational games in class as a means to enhance motivation and engagement in mathematics lessons?*

To address these questions, the following objectives were defined:

- To explore strategies for incorporating Open Middle Problems into a GBL methodology in mathematics education;
- To examine students' perceptions of the game's use in the classroom, particularly its impact on their motivation and engagement in learning mathematics.

A qualitative case study approach (Yin, 1994) was employed to explore the educational potential of the game. The 44 participants were undergraduate students enrolled in mathematics education courses at the University of Aveiro and the Polytechnic Institute of Viseu, in Portugal.

The game was implemented in some classes taught by the authors that follow a consistent structure: brief explanation, game play with time limits, and scoring, often accompanied by informal incentives to encourage participation.

Data collection was carried out through inquiry techniques and direct observation. Two instruments were used: (1) notes taken during those classes, documenting student engagement,

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collaboration, and general classroom dynamics; and (2) a questionnaire completed by 44 students at the end of the semester via Google Forms.

The Open Middle Game

The game presented in this paper differs from traditional mathematical competitions, such as the Mathematical Olympiads, Mathematical Kangaroo, or Martin Gardner-style puzzles. Instead, it is part of a collection of games specifically designed with educational objectives in mind and closely aligned with the school curriculum. This approach follows the principles set forth by Paola Morando and Maria Luisa Spreafico, who have developed numerous games within this framework (La Fortuna et al., 2022; Morando & Spreafico, 2023; Morando & Turconi, 2022).

A distinctive feature of these games is the element of randomness, ensuring that even the most mathematically proficient students do not automatically win. This unpredictability fosters an inclusive and engaging classroom environment, where all students have a fair chance to succeed. Furthermore, these games are structured for group participation rather than individual competition, reinforcing the importance of collaboration.

For a game to be an effective learning tool, several essential principles should be followed (Kapp, 2012). In particular, we highlight the following:

- Universal Participation: The game should be structured so that all students play simultaneously, maximizing engagement and involvement.
- Incentives: Incorporating small rewards – such as candy, recognition in the school newsletter, chocolates, or even homework exemptions – can motivate students and increase their enthusiasm.
- Penalty Mechanisms: To maintain fairness, games should include penalties for unanswered or random guesses, ensuring that students are encouraged to think critically before responding.
- Clear and Simple Rules: The game should be easy to understand and follow, preventing unnecessary confusion.
- Balanced Pace and Difficulty: The game should be neither too easy nor too difficult, ensuring that it remains challenging yet achievable.
- Fun Factor: Ultimately, a game is only effective if it is enjoyable. If it does not provide an engaging and fun experience, it is unlikely to have a lasting impact on learning.

By integrating these principles, mathematical games can transform traditional classroom activities into interactive, engaging, and effective learning experiences.

The game proposed in this paper embodies these principles by fostering critical thinking, strategic reasoning, and collaborative problem-solving, while at the same time incorporating chance elements that balance the opportunity to win.

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The Open Middle Game is a non-digital mathematics-based game inspired by Open Middle problems. It is designed for 2 to 4 teams, with 1 to 4 players per team.

Materials Needed

- Deck of 24 "luck" cards which determine specific game conditions:
    - Fifteen cards say "Pick a card for all teams"
    - Three cards say "Pick a card just for you team" (with a smiling emoji)
    - Three cards say "Pick a card just for the other teams" (with a crying emoji)
    - Two card say "Earn one point" (with a luck symbol)
    - One card says "Loose one point" (with an angry emoji)
- Deck of challenge cards:
    - each card presents a math problem with solutions on the back.
    - If the time required for solving the problem varies significantly from problem to problem, then each card can also present a time limit.
- A stopwatch to limit the time for solving each challenge.
- Paper and pencils (for calculations).
- Beans or tokens (used as points).

Figure 1 shows examples of luck cards.

Pick a card, just for your team	Pick a card, just for the other teams	WIN 1 POINT	LOOSE 1 POINT
Pick a card for everybody	Pick a card for everybody	Pick a card for everybody	Pick a card for everybody

Figure 1. Examples of luck cards.

Figure 2 shows examples of challenges operations with natural numbers. In each case, students must choose digits from the bottom row of the card – without repeating any – to fill in the blank spaces, while satisfying the condition stated at the top.

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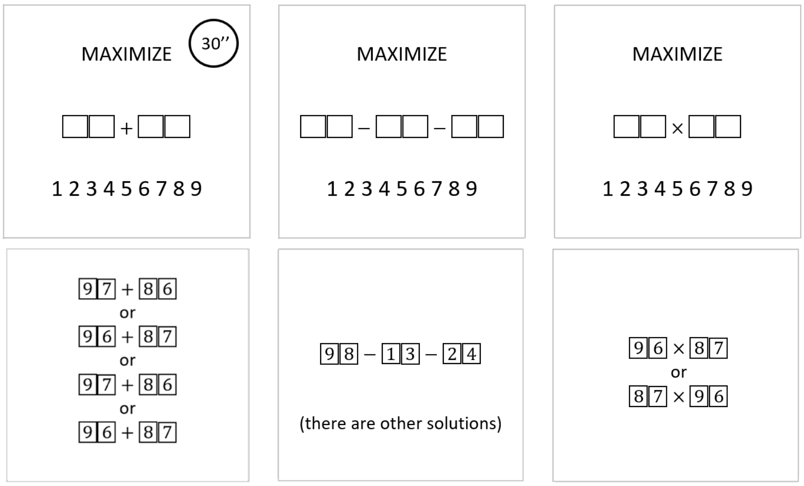

Figure 2. Examples of challenge cards (first row – challenges; second row – solutions).

How to Play

1. Each team takes turns drawing a "luck" card.
2. If the card allows play, a challenge card is drawn.
3. Players must solve the mathematical challenge within the given time (default is 1 minute).
4. They must place digits in the correct spots following the challenge's conditions, ensuring they do not repeat digits unless allowed. Digits are usually chosen with the sets 1 to 9 or 0 to 9.
5. At the end of the time, the card is flipped to reveal the solution.
6. Teams that correctly solve the challenge earn one point.
7. The game continues until either the allotted time for the entire game ends or the deck of challenge cards ends.
8. The team with the most points wins.

Educational Benefits

This game encourages students to explore different solution paths, allowing them to approach problems in diverse ways. By engaging in creative and flexible problem-solving, students strengthen their conceptual understanding of mathematics while developing a deeper appreciation for its underlying principles. The game also fosters rich mathematical discussions, as players share their reasoning and compare strategies, promoting a collaborative learning environment. Additionally,

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it enhances critical thinking skills, encouraging students to analyze problems carefully and make strategic decisions. By integrating problem-solving into an engaging and competitive classroom activity, this game creates a dynamic and interactive learning experience that makes mathematics a subject that students can actively enjoy rather than fear.

Results and Discussion

We have developed multiple challenge decks for the Open Middle Game, each targeting different mathematical topics, namely:

- Addition and subtraction of natural numbers
- Multiplication and division of natural numbers
- Operations with decimals
- Operations with fractions
- Operations with mixed numbers
- Operations with time
- Divisibility rules
- Differentiation
- Integration

Each deck contains 18 to 36 challenges, depending on the complexity and time required to solve them.

To optimize learning, each game session should focus on a limited number of concepts. This targeted approach allows for better systematization, enabling students to develop a deeper understanding of key mathematical ideas. Instead of introducing multiple topics in a single session, it is more effective to play several rounds, each dedicated to a specific concept. This strategy helps prevent cognitive overload and promotes a structured learning experience. Rather than diversifying the subject matter, variation should come from adjusting the difficulty level and types of challenges, while keeping the number of concepts concise and focused.

To illustrate the structure and content of the game, we have selected a sample of challenges. We describe these challenges in detail, highlighting the reasoning processes they encourage and the key mathematical concepts students are expected to learn through them.

The first examples, shown in Figure 2, relate to the first two decks, dealing with natural numbers. In all three challenges, players must fill the boxes with digits (1 to 9, each used at most once) to satisfy a given condition.

The challenge on the left is a maximization problem involving addition (maximize the sum of two two-digit numbers). This task emphasizes place value, helping students understand the importance of assigning the highest digits to the highest place value positions (i.e., the tens). Some students quickly recognize the need to select the largest digits but overlook the significance

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of place value. As a result, they might incorrectly choose 98 + 76. More often than not, upon seeing the correct solution, they immediately realize their mistake. Still within this example, the concept of commutativity is also explored. As shown in the solution in Figure 2, there are four equivalent answers that differ only in the order of the terms or in the arrangement of the tens and units digits. Commutativity is a fundamental property of addition, and this challenge allows students to observe it in practice.

Since the deck contains multiple challenges centered on maximizing addition, with variations in the number of digits per term and the number of terms in the sum, students have several opportunities to refine their reasoning and apply the correct strategy as the game progresses.

The central challenge in Figure 2 is a maximization problem involving subtraction. To maximize a difference, the minuend should be as large as possible, while the subtrahends should be as small as possible. In this particular example, the subtrahend is broken into two parts (two consecutive subtractions), so the first term should be the maximum – 98, with 9 placed in the tens position. The other two terms should be as small as possible – 13 and 24, or other combinations where 1 and 2 occupy the tens place and 3 and 4 the units. As in the previous example, place value plays a key role, and commutativity is also relevant.

The right-most challenge of Figure 2 is similar to the first example but involves multiplication. At first glance, students might think that there are four solutions as in the addition case, but that is not the case. Multiplying two digit numbers involves not only multiplication but also addition since each number is the sum of two quantities, units and tens. Therefore, the distributive property becomes relevant, making the reasoning more complex than in the case of addition – 96x87=90x80+**90x7**+**6x80**+6x7 which is greater than 97x86=90x80+**90x6**+**7x80**+7x6. While commutativity still plays a role in this challenge, its effect is less significant than in the addition example.

All challenges in Figure 3 are optimization problems. The one on the left involves addition and may take more time to solve than the previous examples, as it requires actual calculations rather than relying solely on reasoning about digit placement. This sum, involving three addends, necessarily requires regrouping in both the units and tens columns. Solving it demands strong familiarity with addition in the base-10 system and offers a good opportunity to practice mental arithmetic.

In this specific optimization challenge, it is possible to reach the exact target sum. In other optimization challenges, an exact match may not be achievable, and the goal becomes finding the closest possible value, introducing an extra layer of uncertainty and complexity to the task. The three remaining examples in Figure 3 illustrate this situation.

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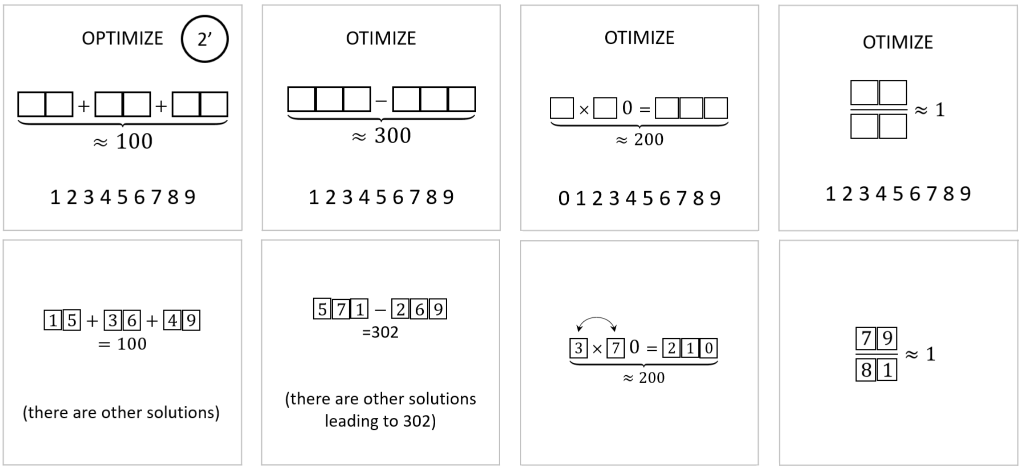

Figure 3. Examples of optimization challenge cards (first row – challenges; second row – solutions).

The second-from-the-left example involves a subtraction problem. Because digit repetition is not allowed, achieving the smallest possible difference requires placing 1 and 9 in the units digits, resulting in a minimum distance of two units from the target value.

The third example in Figure 3 involves multiplication. Here again, due to the restriction on repeating digits, the target value (200) cannot be reached. Note that a 0 must be placed in the units digit, since one of the factors ends in zero. This prompts students to search for a product in the multiplication tables closest to – but not equal to – 20, leading them to choose 3 × 7 = 21.

The fourth example involves fractions, where the target value is 1. Since digit repetition is not allowed, the value 1 cannot be achieved exactly. Students must instead find a pair of numbers, for the numerator and denominator, that are as close as possible to each other. As in the previous example, a difference of two is required, placing 1 and 9 in the units digits. In addition, the denominator should be as large as possible, minimizing the impact of the difference on the value of the fraction. Due to these constraints, there is only one valid solution for this challenge.

Figure 4 presents four additional examples, covering operations with decimals, mixed numbers, and time.

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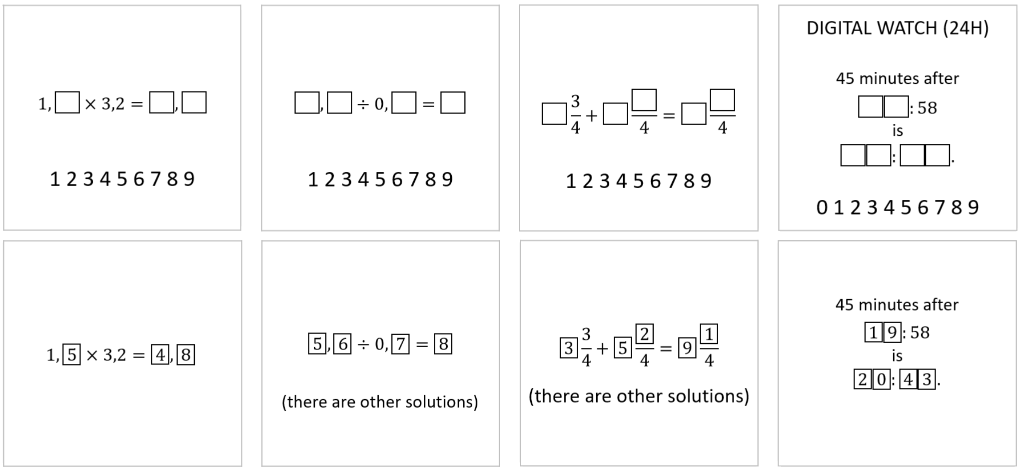

Figure 4. Examples of challenge cards on decimals, mixed numbers and time (first row – challenges; second row – solutions).

The first two challenges focus on multiplication and division with decimals. These tasks require students to have a solid understanding of decimal placement – a skill that is often under-developed due to the widespread use of calculators when working with decimals.

In the left-most example, the digits after the decimal point must be chosen so that their product ends in zero. However, since the final answer is displayed with only one decimal place, the trailing zero is not visible. The only possible pair that satisfies this condition is 5 × 2, and because 2 is already present in the second factor, the choice becomes constrained. The rest of the challenge then relies on accurate calculation.

The second example is essentially a division of integers, though this may not be immediately apparent. If students multiply both the dividend and the divisor by 10, the relationship becomes clear. However, it is not necessary to recognize this transformation. Instead, students can determine the number of decimal places in the quotient by subtracting the total number of decimal places in the dividend and divisor. This challenge has multiple possible solutions: any simple product from the multiplication tables resulting in a two-digit number (with no repeating digits) will work.

In the third example, students must understand the concept of mixed numbers and recognize that all valid solutions require regrouping the fractional parts, resulting in an additional unit being added to the whole number part of the sum. It is also important to observe that the numerator of the second fraction cannot be 1, as the final result must include a fractional part.

The fourth challenge involves time calculations. At first glance, multiple solutions may seem possible, but due to the restriction on digit repetition, there is actually only one valid solution. Adding 45 minutes to any given time can result in either remaining within the same hour or advancing to the next. However, since no digits may be repeated, the sum must cross into the next hour. The only pair of consecutive hours that do not share any digits are 19 and 20. All other adjacent hours share at least one digit, making them invalid under the constraints of the challenge.

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Figure 5 illustrates challenges involving integration and differentiation.

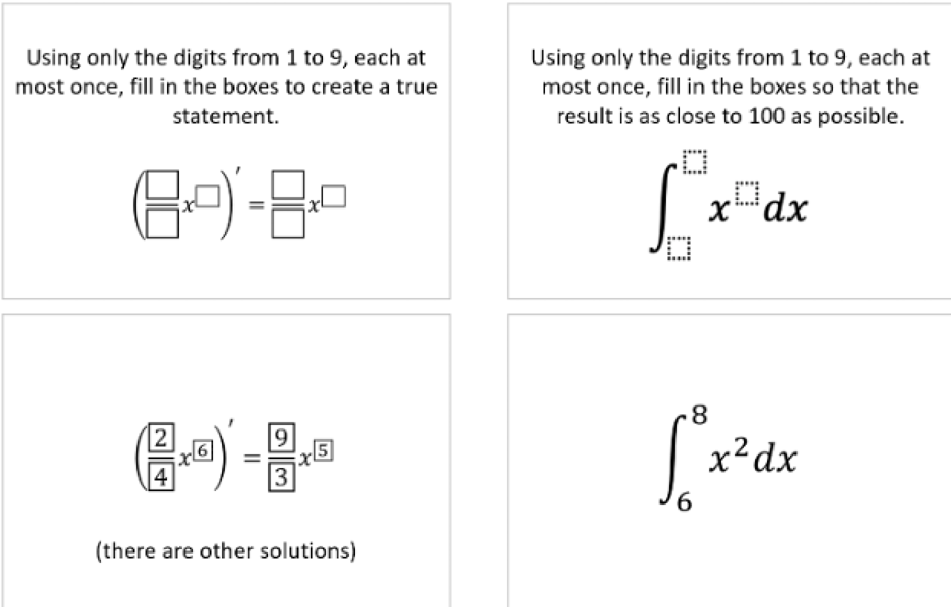

Figure 5. Examples of challenge cards on differentiation and integration
(first row – challenges; second row – solutions).

The top-left side of Figure 5 is one example of a challenge involving differentiation.

To solve this challenge, the students can use, for example, one of the following different approaches:

- Trial and Error with Constraints: give different values to each box and verify if the equation is true, which could take too much time.
- Understand that the derivative of $\frac{a}{b}x^n$ is $\frac{a \cdot n}{b}x^{n-1}$ and that, to get a solution, it must match the right side, $\frac{c}{d}x^m$. Consequently, this means two things must hold: the exponents must satisfy $m = n - 1$, and the coefficients must satisfy $\frac{a \cdot n}{b} = \frac{c}{d}$. From that point it's just necessary to calculate a, b, c, d, n, m under constrains.
- List possible values for n (e.g., 2 through 8, since must also be a positive integer). For each n, iterate through possible a and b values, compute $\frac{a \cdot n}{b}$ and check if it matches $\frac{c}{d}$ with the remaining digits.

This problem is a great way for students to sharpen their calculus skills, especially differentiation, while also getting a good workout in algebra and logical thinking. They need to be careful when picking digits, making sure none are used more than once and that the math equation still holds true. It's all about exploring and being precise, because even a tiny switch

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in where a digit goes can completely change the answer or lead to a mistake. Basically, this challenge mixes calculus, basic math, and problem-solving, pushing students to really think outside the box and make sure they understand the core math ideas.

Now, for the rightmost challenge in Figure 5, students really need to know their integral calculus, especially how to figure out definite integrals of power functions. They also need to be good at planning and solving problems, because they'll have to try out different combinations of digits for the top and bottom numbers of the integral and the exponent, all while making sure they don't use any digit twice.

There are a few ways students can tackle this. One way is to pick an exponent first and then try out different pairs of limits to see which combo gets them closest to 100. But that could take a while! Another idea is to kind of work backward: think about what the limits and exponent would roughly need to be to get an answer around 100, and then fine-tune those choices. They could also use some algebra smarts to narrow things down – for example, realizing that smaller exponents are probably better because bigger ones make the result grow super-fast, making it hard to stay close to 100. Also, limits that are closer together will help keep the integral's value smaller. Talking and working with classmates can also be super helpful, as sharing ideas might lead to quicker ways to solve it or solutions they might have missed. All in all, this challenge doesn't just test their math skills; it also helps them stick with a problem and think creatively to find solutions.

The Open Middle Games have been implemented in the classroom across several courses taught by the authors of this paper.

Figure 6 shows examples of students engaging with the game during class. The photos show students solving challenges on paper while using the timer function on their mobile phones. Beans were used as scoring tokens, and at the end of each session, sweets were awarded to the winning teams.

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Figure 6. Photos of undergraduate students playing the Open Middle Game during math classes.

These students were pre-service teachers at the undergraduate level, who played five different versions of the game over the course of one semester. They were organized into pairs or small teams of two students each, and played with multiple copies of the game.

At the end of the semester, students completed a questionnaire used to assess students' perceptions regarding the use of educational games in the classroom. The questionnaire was answered by 44 students.

Participants were asked if they liked playing educational games in the classes, through a five-point Likert scale. As Figure 7 shows, there were no negative responses.

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Figure 7. Pie chart of responses.

Further on, students answered some questions concerning educational games in mathematics classes. The responses are summarized in Figure 8.

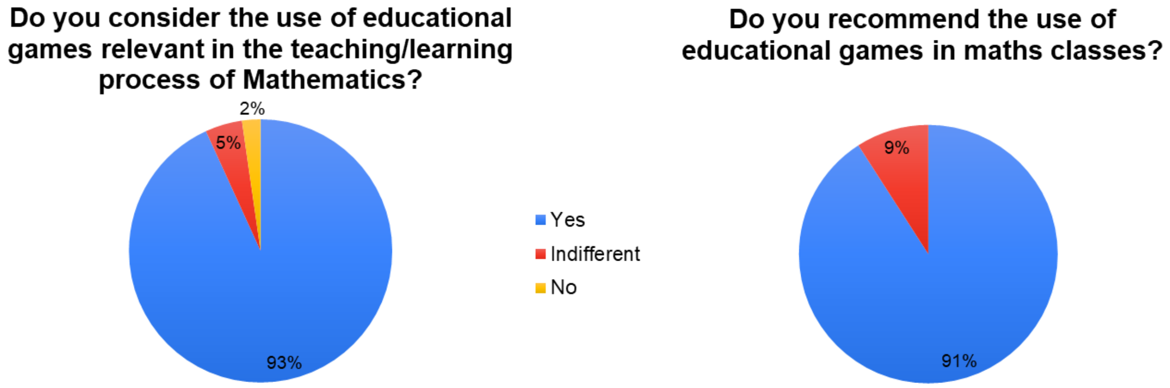

Figure 8. Pie charts of responses.

Figure 8 indicates strong student agreement that games are a valuable resource in the teaching/learning process. Additionally, students recommend using educational games in mathematics classes.

The heatmap in Figure 9 presents the results of a Likert-scale questionnaire in which students rated the extent to which educational games contributed to various aspects of their mathematics learning experience (1 = Strongly Disagree; 5 = Strongly Agree). The data shows a consistently high level of agreement across all six statements, with mean scores above 4,0 and low standard deviations, indicating both positive perceptions and consensus among students (note that one student did not answer this question).

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The use of educational games in maths classes contributed to:	1	2	3	4	5	Mean	SD
making learning more interesting.	0	1	4	14	24	4,4	0,8
making learning more interactive.	0	0	4	9	30	4,6	0,7
making classes more stimulating.	0	0	6	13	24	4,4	0,7
involving the students in the learning process.	0	1	2	15	25	4,5	0,7
increasing student motivation.	0	3	5	14	21	4,2	0,9
a more positive view of mathematics	0	0	8	17	18	4,2	0,8

Figure 9. Heatmap of responses related to the use of games in maths classes.

The strongest areas of impact were on making learning more interactive (mean = 4,6) and on involving students in the learning process (mean = 4,5). These results suggest that the inclusion of games significantly enhanced student engagement and participation, aligning well with the goals of active learning methodologies.

The lowest (yet still positive) mean score was for “increasing student motivation” (mean = 4,2; SD = 0,9), which may reflect individual differences in how students respond to gamified environments. Nonetheless, even in this category, responses clustered in the higher end of the scale, with few students expressing disagreement.

Overall, the heatmap provides strong evidence that non-digital educational games – such as the Open Middle Game – effectively support student motivation, foster positive attitudes toward mathematics, and contribute to a more dynamic and inclusive classroom experience.

Conclusions

This paper presents an educational game designed to promote critical thinking, problem-solving, and collaborative learning, inspired by the philosophy of Open Middle Math Problems. The authors developed multiple versions of the game, tailored to various mathematical topics ranging from elementary arithmetic to undergraduate-level calculus.

The study aimed to examine students’ perceptions of the game’s use in the classroom, particularly regarding its impact on their motivation and engagement in learning mathematics. To achieve this, several versions of the game were implemented in classroom settings, and data were collected. In addition to the authors’ observations during the game’s application, a survey was conducted at the end of one of the courses.

The analysis of the responses indicated that the use of non-digital educational games significantly enhanced students’ motivation and engagement in learning mathematics. Students not only reported enjoying the games but also felt more actively involved in the learning process. These findings were consistent across both the quantitative survey results and qualitative classroom observations.

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The games served a dual purpose: reinforcing mathematical understanding and equipping pre-service teachers with practical resources for their future classrooms. The positive reception of the games was nearly unanimous – none of the students expressed dislike, and many recommended their continued use in math education.

These findings reinforce the idea that well-designed educational games – particularly non-digital ones like the Open Middle Game – can serve as a meaningful bridge between curriculum goals and student engagement. The challenges embedded in the game not only promote procedural fluency and conceptual understanding but also create opportunities for productive struggle, peer discussion, and strategy sharing. The open nature of the problems encourages flexibility in thinking and supports the development of metacognitive skills as students reflect on their approaches.

Moreover, this study shows that Open Middle Problems can be successfully incorporated into a game-based learning methodology by transforming them into structured, time-bound challenges that maintain the core principles of the Open Middle framework – fixed start and end points with multiple solution paths – while adding elements of chance, competition, and collaboration. The game format encourages diverse strategies and promotes engagement through immediate feedback and team dynamics, making it an ideal vehicle for bringing the richness of Open Middle thinking into a playful, interactive setting.

Furthermore, the collaborative and game-based environment provides an alternative to traditional assessment and instruction, shifting the focus from right answers to mathematical reasoning. This helps foster a classroom culture in which error is normalized, risk-taking is encouraged, and persistence is rewarded. The game's balance between structure and freedom – clear constraints combined with multiple solution paths – makes it a powerful tool for inclusive mathematics teaching.

In addition to the cognitive and social benefits observed in gameplay, the Open Middle Game offers a valuable professional learning experience for pre-service teachers. By engaging with the game as learners, they are exposed to pedagogical strategies they can adapt and implement in their future classrooms, promoting innovation in mathematics instruction.

In summary, the research questions that guided this study were answered positively. The Open Middle Game proved to be not only enjoyable and effective, as supported by previous research (Hall et al., 2024), but also pedagogically sound, versatile, and easily adaptable to a wide range of mathematical topics and student levels. Its use in teacher education contexts is particularly promising, as it simultaneously enhances student motivation and fosters greater engagement in the learning process.

Authors contributions

Conceptualization: Nuno Bastos and Andreia Hall; Methodology: Nuno Bastos; Software: N/A; Validation: N/A; Formal analysis: Nuno Bastos and Andreia Hall; Investigation: Nuno Bastos and Andreia Hall; Resources: Nuno Bastos and Andreia Hall; Data curation: N/A; Writing – original draft: Nuno Bastos and Andreia Hall; Writing – review and editing: Nuno Bastos and Andreia Hall; Visualization: Nuno Bastos and Andreia Hall; Supervision: N/A; Project administration: N/A; Funding acquisition: N/A

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