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Observations about engaging with historical mathematical problems with help of ChatGPT for classroom work in schools and undergraduate courses

Observações sobre o envolvimento em problemas matemáticos históricos com a ajuda do ChatGPT para trabalho em sala de aula em escolas e cursos de graduação

Observaciones sobre la resolución de problemas matemáticos históricos con ayuda de ChatGPT para el trabajo en el aula en escuelas y cursos de licenciatura

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Abstract

Currently, there is great interest in incorporating the use of Artificial Intelligence with ChatGPT version 3.5 (AI) into day-to-day school academic work. It has been used, for example, in the search for information, in the processing of large volumes of data to carry out tests and calculations. We report here, however, some risks when exploring several topics that involve classic problems, an invaluable source of educational resources for the classroom. Based on this, we propose a non-exhaustive list of considerations for classroom work in schools and undergraduate courses with this computing resource, as well as some development routes for the improvement of AI.

Keywords: Artificial Intelligence with ChatGPT version 3.5 (AI); History of Mathematics (HM); Visualization in Mathematics; calculators with CAS; Dynamic Geometry.

Resumo

Atualmente, há grande interesse em incorporar o uso da Inteligência Artificial com ChatGPT versão 3.5 (IA) no cotidiano escolar. Tem sido utilizado, por exemplo, na busca de informações, no processamento de grandes volumes de dados para realização de testes



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e em cálculos. Contudo, relatamos aqui alguns riscos ao explorarmos, com a IA, alguns temas que envolvem problemas clássicos da História da Matemática (HM), que são fonte inestimável de recursos educacionais para a sala de aula. A partir do exposto, propomos uma lista não exaustiva de considerações para o trabalho em sala de aula com esse recurso computacional, bem como alguns caminhos de desenvolvimento para a melhoria dos processos da IA.

Palavras Chave: Inteligência Artificial com ChatGPT versão 3.5 (IA); História da Matemática (HM); Visualização em Matemática; calculadoras com CAS; Geometria Dinâmica.

Resumen

Actualmente existe un gran interés por incorporar el uso de la Inteligencia Artificial con ChatGPT versión 3.5 (IA) a la vida escolar cotidiana. Se ha utilizado, por ejemplo, para buscar información, procesar grandes volúmenes de datos para exámenes y realizar cálculos. Sin embargo, aquí reportamos algunos riesgos al explorar, con IA, algunos temas que involucran problemas clásicos de la Historia de las Matemáticas (HM), los cuales son una fuente invaluable de recursos educativos para el aula. Con base en lo anterior, proponemos una lista no exhaustiva de consideraciones para trabajar en el aula con este recurso computacional, así como algunas vías de desarrollo para mejorar los procesos de IA.

Palabras clave: Inteligencia Artificial con ChatGPT versión 3.5 (AI); Historia de las Matemáticas (HM); Visualización en Matemáticas; calculadoras con CAS; Geometría Dinámica.

Introduction

In the classroom the excitement is increasing due to the incorporation of Artificial Intelligence with ChatGPT version 3.5 (AI). It is a source for students to carry out documented essays, develop creative projects, collect information, delegate arduous calculations to the machine, and build algorithms to solve various problems, among other applications. On the other hand, academics debate whether its use as a component of topics or courses' learning should be allowed or sanctioned. We want to highlight that ChatGPT version 3.5 has no conceptual understanding of the deep ideas in the produced text. Even ChatGPT version 4.0 (paid version) has limitations, but allow access information of some web sites.

It should be emphasized that free versions of ChatGPT or other similar AI's will be used to a greater extent by students. Identifying areas where they can improve will have an impact on classroom computer literacy, that is, they will drive change and reorganization of teachers' and students' classroom practices (Gadanidis et al., 2024). Human thinking, particularly in relation to the development of Mathematics Education, can be supported by technological environments as diverse as ICTs and AI, but it must be taken into account that this can be both a supporting factor and a disturbance factor in the structures of such thinking (Gadanidis, 2017).

As already mentioned, Artificial Intelligence (AI) has a great impact in the education. Al has provided students with personalized learning experiences and teachers with powerful



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tools to propose interesting projects to their students and adapt their teaching methods, as Green-Harper (2023) reports. The same author indicates that along with the advantages of using AI-based learning platforms, such as ChatGPT, there are risks of negative impact. As an advantage, he points out the ability to engage students in a conversational, human-tutor-like manner, as well as its potential to improve learning outcomes by providing students with more efficient and effective ways to learn. On the other hand, there is also concern about the possible negative impact it could have on students, as some experts argue that it could hinder their critical thinking skills and reduce human interaction, which is an essential aspect of learning (Green-Harper, 2023). Reinforcing this idea, this author indicates that some of the challenges are "bias in AI models, lack of personalization, the possibility of errors, and the fear of job loss for teachers" (Green-Harper, 2023).

An inexhaustible source of teachings for all ages is the History of Mathematics (HM) (Romero et al., 2023). The process of building knowledge is a great collaborative task that has been carried out by all cultures, with multiple resources (from clay tablets to the wing of an airplane). Frequently, new pieces of the historical puzzle come to enrich the overview of ancient knowledge, offering increasingly precise ideas of what was known in previous times.

Mohamed et al. (2022) states that "...previous studies have only focused on the use of Al in the fields of engineering, computer science and STEM. Therefore, using this opportunity, lots of research about the maximum use of Al in mathematics education can be done", and in this line of thinking, we propose HM as an open field for the use in research of Al in Mathematics Education.

In our time, the emergence of AI allows searches of extensive databases, from many digitized libraries. Despite this, the HM seems to be a library too large and elusive to compile (and it turns out that it is growing too). In this work, we will explore the fusion between AI and HM, taking as a sample the following four examples that were the focus of the efforts of mathematicians from different eras: the Basel problem, Lagrange multipliers, Cycloid construction and Pythagorean triples.

These problems are presented and also the questions showed to ChatGPT (version 3.5) as well the answers in the original language in which the interactions were raised. Finally, we analyze this interaction with ChatGPT and present some elements that we consider suitable for a better interaction of the AI in the classroom.

Example 1. The Basel problem, Euler's version.

The Basel problem is a research in mathematical analysis with relevance to number theory, and it was first posed by Pietro Mengoli in 1650 and solved by Leonhard Euler in 1734. This problem was named Basel because it's the hometown of Euler. The Basel problem, in Euler's version, can be expressed like this: Calculate the series $\sum_{i=1}^{n} \frac{1}{n^2}$ using algebra and the infinite series of sine: $\sin x = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$ (Boyer, C. B.; 1968; pp. 545).



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In the classroom, this problem has several facets that can be addressed in various mathematics courses at the university level; in Higher Algebra we usually find the calculation of partial sums as a way to approximate the value of the sum; within the context of an Analysis course, we can be interested in the convergence or not of a series and perform different convergence tests to the series of the form $\sum_{n=1}^{\infty} \frac{1}{n^k}$, $k \in \{2,3,4,5,...\}$; in all of them, constructing the Cartesian graph of the sum for values other than k allows us to explore relationships on a context rich in Analytic Geometry. In this context we asked ChatGPT: What is the Basel problem, from which a way to calculate

 π emerges? How did Euler solve the problem?

The interaction with ChatGPT¹ is shown in Figure 1.



Fig. 1. ChatGPT and the Basel problem. Searching for Euler resources. (The remarks in red and blue are ours).

Two aspects stand out in the transcription of the interaction with ChatGPT, and we group them in red and blue; in red box we can read "Euler found a connection between the Basel problem and the Riemann zeta function, denoted by $\varsigma(x)$, which is a complex function. He showed that $\varsigma(2)$ is equal to $\pi^2/6$, which means that the sum of reciprocals of squares of natural numbers converges to $\pi^2/6$. This was a significant mathematical result and is known as the Euler's solution to the Basel problem". In a blue box we can read "(Anonymous) But Euler died before Riemann born, how can he knows the Riemann zeta function. ChatGPT. ChatGPT. You are absolutely correct, and I apologize the mistake in my previous responses. Leonard Euler indeed lived before Bernard Riemann, and he could not have known about the Riemann zeta function because it was developed by Riemann in the 19th century, long after Euler's time".

¹ Link in: https://chat.openai.com/share/3ec2fec7-d62f-44bc-b3bd-098cd46d643d



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ChatGPT proposes that Euler used techniques that belong to a mathematician who was not yet born, Riemann (Boyer, 1968). ChatGPT recognize its error and give us an apologize for the inconvenience of its answer. Probably this is a good opportunity for a benchmark for ChatGPT learning.

Example 2. Construction of the cycloid.

A very important curve in the HM is the Cycloid, that it's a curve generated by a rolling curve on another base curve. The Cycloid is explained as the locus of a point on a circle that rolls without bouncing on a straight line (Boyer, 1963; IREM Historie des Mathématiques, 1998). For example, and between other properties, the cycloid, with the cusps pointing upward, is the curve of fastest descent under uniform gravity.

A skill that we can promote in the classroom is the ability to create geometric models of phenomena, a skill that is highly valued in areas of physics and various engineering, such as mechanical, civil, biomedical, mechatronics, among others, where geometry courses are reduced to analytical geometry without touching synthetic geometry.

Geometric modelling in the classroom can follow a parallel course to the historical one, where analytic geometry and curves obtained by rolling (les roulettes) led to a renaissance of Greek geometry, which was limited to the construction of curves with the classical restrictions of doing it only with a ruler and compass. The universe of bearing curves and mechanical devices (the piston, connecting rod and crank system is a possible example) are starting points to encourage students to design models not only in the first years of physics, but also in those of algebra, calculus and, of course, analytical geometry, working on examples in contexts closer to their specialties.

Several methods of calculating gear velocities and calculating the quadrature of bearing curves were discovered during the period prior to the creation of differential and integral calculus. These methods have had such an influence on mechanical engineering, for example, that it is often at the time of specialty courses that are presented to students who are trained in this area.

About this theme we inquire to ChatGPT: What is a cycloid? Give a description of its geometric (non-analytical) construction step by step with Cabri.

In Figure 2 it is possible to see the interaction with ChatGPT² in relation to this challenge.

² Link in: https://chat.openai.com/share/f13a3d44-3817-4c22-89bb-e162606fce49



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(The remark in red was made for us).

The transcription of the interaction with ChatGPT do not show how we can coordinate the roll on the wheel circle and the translation in the horizontal sense. In addition, there is not a "Transform Object" in any Cabri Menu. And, how do you get the circle to roll without slipping horizontally? The remark in the red box show us the end of the question for ChatGPT.

ChatGPT do not obtain the Cycloid because it does not use geometrical properties in its construction. ChatGPT know well the drag mode, because it suggest the user drag by self the circle and trace points of the Cycloid, but ChatGPT do not offers any figure or geometric simulation in its interaction. In addition, ChatGPT have no idea about the commands in Cabri.

Example 3. Find five numbers: Lagrange multipliers.

In mathematical optimization, the method of Lagrange multipliers is a method for finding the local maximum and minimum of a function subject to several functions of constraints, and it is



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named due to the mathematician Joseph-Louis Lagrange (1736-1813). In many scientific disciplines arises the problem of optimizing a function of several variables under a series of constraint functions (Gutiérrez & Quintas, 2016). The basic idea is to convert a constrained problem into a form such that the derivative test of an unconstrained problem can still be applied. Lagrange multipliers is a methodology based on the observation of the relation between the gradients' vectors of this functions, introducing as many constants as there are constraint functions in the problem. The great advantage of this method is that it allows the optimization to be solved without explicit parameterization in terms of the constraints. As a result, the method of Lagrange multipliers is widely used to solve challenging constrained optimization problems and it is "an elegant algorithm" (Boyer, 1968).

Constrained optimization problems also appear at different times in the classroom. In the case of having a single constraint and in order to expand the universe of possible examples, we can present the pair objective function and constraint function as the first problem and then talk about the so-called "dual" problem, where these functions exchange their roles. That is, what used to be an objective function now becomes a constraint function and vice versa, if the first problem is to obtain a maximum, the dual problem will be the problem of obtaining a minimum, if the problem refers to obtaining a minimum, the dual will now correspond to obtaining a maximum.

In particular cases, there are calculation alternatives that offer shortcuts that can be presented in the classroom so that the student acquires skills and good judgment when proposing, solving and studying the solutions to these problems. Several calculus resources of this nature are presented for these particular cases on Perelman, (1969) and in such a way that they can be taken up again in university classrooms for the first years of Algebra and Calculus.

In particular, Lagrange multipliers is a methodology that we can use to solve this problem: *Find five positive numbers whose sum is fixed* (as an example for ChatGPT, we gave 57) *and whose product is the greatest possible.*

Figure 3 shows (in original language) some highlights of the interaction with ChatGPT³ about this topic.

EU	Explica como encontrar cinco números positivos cuya suma sea igual a 57 y cuyo producto sea el máximo posible mediante multiplicadores de Lagrange,	ß	g(x1, x2, x3, x4, x5) = x1 + x2 + x3 + x4 + x5 - 57 Ahora, usamos el multiplicador de Lagrange à para encontrar los valores óptimos de x1, x2, x3, x4, y x5, Esto se hace al resolver el sistema de ecuaciones:
	Para encontrar cinco números positivos cuya suma sea 57 y cuyo producto sea máxin podemos usar el método de multiplicadores de Lagrange. Este método nos permite	mo,	∂/∂xi (f - λg) = 0, para i = 1, 2, 3, 4, 5 g(x1, x2, x3, x4, x5) = 0
	encontrar el máximo (o mínimo) de una función sujeta a una restricción.		Después de algunos cálculos, encontramos que los cinco números que maximizan el
	Primero, definimos nuestra función objetivo como el producto de los cinco números:		producto son 9, 9, 9, 15 y 15. Estos números tienen una suma de 57 y un producto de 91,125.
	f(x1, x2, x3, x4, x5) = x1 * x2 * x3 * x4 * x5		Por lo tanto. los cinco números positivos cuva suma es 57 y cuvo producto es máximo
	Luego, establecemos nuestra restricción como la suma de los cinco números:		son: 9, 9, 9, 15 y 15.

Fig. 3. ChatGPT and Lagrange multipliers. Testing calculation power. (The remark in red is our)

The translation of the remark in the red box is *"Therefore, the five positive numbers which sum is 57 and its product is a maximum are 9, 9, 9, 15 and 15."*. ChatGPT do not show its calculus.

³ The complete interaction is in the following link (it has translation to english too): https://chat.openai.com/ share/2a5c9794-2fe8-4fe7-acff-f346f66094d3



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The solution is that all numbers could be equal to $\frac{57}{5}$. ChatGPT do not consider the possibility of all numbers could be equal, may have restricted the solution only to positive integers. Of course, there is not any refer to its dual problem or other related problem to help us in the calculus.

Example 4. Pythagorean triples.

A problem as old as the search for certain right triangles and that is renewed by the approach with matrices is the calculation of the Pythagorean triples.

A triplet of positive integers numbers a, b and c forms a Pythagorean triplet if it satisfies one of three conditions: $a^2 + b^2 = c^2$ or $a^2 + c^2 = b^2$ or $b^2 + c^2 = a^2$. The triplet is primitive if and only if the greatest common divisor of a, b and c is 1 (Boyer, 1968; Hall, 2009; Roberts, 1977).

In the classroom we may find the belief that "everything" is already written within Mathematics; that it is believed that ancient knowledge "has already been superseded"; that there is "nothing new" to discover; that the old "is already tedious." And so, then, the calculation of the Pythagorean triples, at a distance of more than 30 centuries, what could it offer us?

About this we inquire to ChatGPT: What is a Pythagorean triple? What is the oldest known method to calculate them? How can they be calculated using multiplication of matrices of size 3x3?

For this example, the highlights of the interaction with ChatGPT⁴ are shown in the Figure 4a and Figure 4b.

I'd be happy to translate the conversation into English for you:	This new vector '(0, 1, 0)' represents the vector '(b, -a, c)'.
User: Explain how to generate a Pythagorean triple using a 3×3 matrix multiplication.	You can continue multiplying matrix "A" with this new vector to obtain the vector "(0, 0, 1)", which represents "(c, b, -a)".
Assistant: To generate a Pythagorean triple using 3×3 matrix multiplication, you can use the following matrix:	
	c = [[0, -2, 0], [1, 0, 0], [0, 0, 1]] * [0, 1, 0] = [0,
& = ((0, -4, 0), (1, 0, 8), (0, 0, 1)) This matrix is known as the rotation matrix in the xy-plane, which rotates a point by 90 degrees counterclockwise around the origin. You can use this matrix to generate a Pythagorean triple by multiplying it with the vector `(1, 0, 0)`, which represents the vector `(a, b, c)` in the xy-plane, where `a` is the length of the	In this way, you can generate a Pythagorean triple `(a, b, c)` by multiplying matrix `A` with the vectors `(1, 0, 0)`, `(0, 1, 0)`, and `(0, 0, 1)`. User: Describe the Babylonian method for generating Pythagorean triples. Assistant: The Babylonian method for generating Pythagorean
side opposite to the right angle.	triples is an algorithm used to find a Pythagorean triple `(a, b, c) given a number `c`. This method was developed by Babylonian
D [H0, -1, 00, [1, 0, 40], H0, 0, 1]] [1, 0, 0] [0,	mathematicians thousands of years ago and is based on the Pythagorean theorem, which states that for any right triangle, `a^2 + b^2 = c^2`.

Fig. 4a. ChatGPT and Pythagorean triples. Searching for Roberts' theorem and the Babylonian

⁴ The complete interaction is in the following link: https://chat.openai.com/share/094c11e3-91bc-4c74-839a--30d87cc76ac1



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In the Figure 4a, we can see that ChatGPT do not identify the Roberts' theorem, however it recognizes the Pythagorean triples and try to find the Babylonian method, but let's see the following interaction in Figure 4b.



Fig. 4b. ChatGPT and Pythagorean triples. Building calculation algorithms. The remark show us ChatGPT give an iterative method which it is different to the Babylonian method. (The remark in red is our)

The remark in the red box: "This process is repeated until a Pythagorean triple is found". Typically, it is being suggested here that what has been described constitutes an iterative method that is moving towards a solution that could eventually take several steps.

This interaction show us that ChatGPT give an iterative method, but not the Babylonian method. For this topic, two references that were not found by ChatGPT are central, in historical terms, and they are the Plimpton tablet 322 (Figure 5) and Roberts' theorem (Figure 6).

The historical interpretation of this information is well known, for example, in Neugebauer (1957) and Boyer (1968).



Fig. 5. Plimpton Tablet 322 with the first Pythagorean triples calculated by an anonymous author according to (Boyer, 1968) - reference not found by ChatGPT.

ChatGPT do not consider a classical book of HM as (Boyer, 1968). So, the question remains as to why ChatGPT did not consider the original references in its searches. It is an opportunity to expand ChatGPT.



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15. A triple of integers x, y, z for which x"+y"=z" is called a Pythagorean triple. when the integers have greatest common divisor 1 we call the triple primitive. A triangle whose side lengths form such a triple is called a Pythagorean triangle. It is clear that all Pythagorean triples are integral multiples of primitive triples. Define the three matrices U,A,D by : $U = \begin{pmatrix} 1 & 2 & 2 \\ -2 & -1 & -2 \\ 2 & 2 & 3 \end{pmatrix}, A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 3 \end{pmatrix}, D = \begin{pmatrix} -1 & -2 & -2 \\ 2 & 1 & 2 \\ 2 & 2 & 3 \end{pmatrix}.$ show that (x',y',z') is a primitive Pythaqorean triple if and only if (x',y',z')=(3,4,5) & , where A is a finite product of matrices each factor of which is one of U, A, D. I.e. show that every Pythagorean triple is in the following array where the lines leading to the right from any triple correspond to applying to that triple either the matrix U (for up), A (for across) or D (for down).

Fig. 6. Roberts theorem according to Roberts (1977) - reference not found by ChatGPT.

ChatGPT do not consider the Roberts' research, a beginner in this area. Maybe it is a very old reference. Neither it does not consider (Weisstein, E., 1998). In addition, ChatGPT does not refers the fixed graphs from Wikipedia⁵.

This is another great ensemble of references to consider in the future to ChatGPT.

Analysis of interactions with ChatGPT

In the Basel problem, ChatGPT proposes that Euler used mathematical resources from Riemman, a mathematician of a later era (Boyer, 1968). This indicates that ChatGPT could generate texts that distorting the HM. So, an inexperienced student may take his version of events as true if he is not supported by the teacher or expert peer.

In the proposed solution to the Lagrange multiplier problem, ChatGPT does not mention that the selected numbers can all be the same (the correct answer) and focuses on non-optimal solutions. This indicates that ChatGPT is not using computing power or that it is limited. And again, an inexperienced student may take his version of events as true.

The problem of constructing the Cycloid has the restriction of using only geometric tools (Díaz Barriga, 2014). We focus on Cabri, one of the 3 tools that ChatGPT recognizes as the most powerful for Dynamic Geometry. The interaction shows that ChatGPT proposes a construction procedure, which is incorrect for two reasons: it does not know the interface tools and it does not rely on geometric properties. On the other hand, its procedure does not promote step-by-step visualization of what it proposes.

The construction of the Cycloid strongly promoted new curves in Mathematics. In Analytic Geometry in particular they were promoted by French mathematicians. It is interesting to note that

⁵ Pytagorean triplets fixed graphs are in the following link: https://es.wikipedia.org/wiki/terna_pitag%c3%b3ricae



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on commercial products as a packaging box of the TI-92 plus calculator, the construction of the cycloid is illustrated. The Cabri website traditionally appears the animation of the construction of a bicycle and a cycloid associated with the movement of a point on one of its wheels (Figure 7). In the future ChatGPT could consider references like this because commercial products remarks too valuable information indirectly.



Fig. 7. The cycloid in the TI-92 manual (left) and CabriLog portal with the bicycle and the cycloid

Reviewing the interaction on Pythagorean triples, ChatGPT version 3.5, not only confuses the Babylonian method (Boyer, 1968; Stillwell, 2010) with an iterative method but also ignores Roberts' theorem (Roberts, 1977; Weisstein, 1998; Hall, 2009). This indicates that its searches do not reach very basic history books or some specialized references that mark a relatively recent mathematical discovery. ChatGPT was not be able to cross-check the information that comes from them, in the classroom we expect this advance in the future.

As a common aspect to all the problems reviewed is that ChatGPT does not promote any geometric visualization of the problems, which is a very desirable representation element when a student wants to deeply understand Mathematics (Martins, Estrada & Nascimento, 2014).

Some elements that we would like for an ideal AI in the classroom.

In the Basel problem, historical sources (Boyer, C. B.; 1968; pp. 545) indicate that the sums of the form:

$$\sum_{n=1}^{\infty} \frac{1}{n^k}, \ k \in \{2,3,4,5,\dots\}$$

are very interesting for Euler. To this days, solving the cases in which k is odd remains an open problem; Euler solves the cases in which k is even up to the value of k=26. A graphing calculator with



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CAS is capable of calculating this sum at least up to k=80. One would expect that an AI could signal that some problems are open and/or offer a sample of its computing power on historical problems.

The problem of the construction of the cycloid was a source of debate among European mathematicians of the 17th century, particularly in France with Fermat, Pascal and Roberval, not only the curve itself but the problems of drawing its tangent and the calculation of its quadrature (IREM Historie des Mathématique, 1998).

ChatGPT does not resort to historical sources (or probably prefers those in English) nor does it look for information in Dynamic Geometry calculator manuals, which could be very recent sources. Nor did he review publications or research material in Dynamic Geometry, even though the interfaces of Cabri II plus or recently GeoGebra have been in academic discussion for a long time: ChatGPT prefers to invent tools for the interface and does not use geometric properties to model that curve. But ChatGPT could expand in these areas in the near future.

Our last example links very ancient knowledge with modern knowledge; In addition, it can be presented with strong numerical, algebraic, and graphical environments interconnection. For this reason, this example will immediately merit a deeper and more extreme development. Here we focus on its relation to Roberts' theorem and its exploration with dynamic geometry, which gives another very rich environment for mathematics education: patterns with matrices.

The problem of calculating Pythagorean triples unifies two historical moments very distant in time, 1800 B.C. with 1977 A.D.: the calculation with the Babylonian method and the calculation using Roberts' theorem (Boyer, 1968; Hall, 2009; Roberts, 1977; Weisstein, 1998). Crossing the two methods of calculating Pythagorean triples can lead to academic exploration of regular patterns (Figure 8), to visualizing numerical relationships or exploring topics of current mathematical research.



Fig. 8. Four-level tree diagram that illustrates the Pythagorean triples obtained with the Roberts Theorem and the pairs of integers that generate them according to the Babylonian method (calculations without using ChatGPT) - some numerical patterns.



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If the approach of ChatGPT is also strengthened with representations such as those offered by Cabri's interactive geometry or even the one found in the TI-92 plus graphing calculator itself, visualization and mathematical discovery will be promoted in a very concrete way in the classroom (Martins, Estrada & Nascimento, 2014).

ChatGPT searches were expected to retrieve as much information as possible from the multiple access points offered by this problem. ChatGPT version 3.5 did not find the static graphical representations obtained from Wikipedia, to mention a widely consulted source of information, in addition to its lack of knowledge of the Babylonian method and Roberts' Theorem. So, beyond the limitations, let's look at the broad field of work in which progress has to be made to improve a resource that we would like to have an impact on the mathematics classroom.

The calculation of Pythagorean triples has already been addressed by the group of researchers of Gadanidis et al. (2023), to explore interactions between programming, teaching and learning of mathematics. For other proposals, like find the Euler's bricks, an open problem, these kind of resources could be very useful (Costa, A., 2023), because the calculations of triplets is fast and at the hand of the researcher. It should be noted that the calculation scripts for both methods are implemented in a few lines of code (Figure 9). If ChatGPT expands its calculation power in the future, maybe can help to solve this question with examples.

The Cabri Dynamic Geometry environment allows the definition of the macro for a triangle with known sides (it does not matter if they are Pythagorean triples or not); allows too create the macro to implement the calculation of the Pythagorean triple by means of the Babylonian method, from the coordinates of a point that moves on a grid. It is another future area for the calculation power of ChatGPT, not so far of its Python version.

Metodo babilonico para calcular ternas pitagoricas	Script para explorar el teorema de Roberts sobre Ternas Pitagoricas
C: 2+k C: 1+1 C: k^2-1^2+aa C: 2*k*1+bb C: k^2+1^2+cc C: [aa,bb,cc] C: aa^2+x C: bb^2+y C: cc^2+z C: cc^2+z C: x+y-z C: x+y-z C: x+y-z C: y+z-x	C: [3,4,5]+v C: [a,b,c]+w C: [a,b,c]+w C: [[-1,-2,-2][2,1,2][2,2,3]]+u C: [[1,2,2][-2,-1,-2][2,2,3]]+d C: [[1,2,2][2,1,2][2,2,3]]+d C: [[1,2,2][2,1,2][2,2,3]]+d C: [[1,2]]+d C: [[1,2]]+d



Using Dynamic Geometry we can not only calculate the Pythagorean triples with the Babylonian method or with the Roberts theorem, we can also do it with the Fibonacci numbers. But even more: we can know if a triple is Pythagorean or not; we can switch to the geometric register and visualize the respective right triangles; observe regularities when exploring the Pythagorean triple tree; discover certain patterns by using special products in Roberts' theorem that have not been indicated in the original articles (Roberts, J.; 1977; Hall, A.; 2009). We have here a source of problems that can be explored in the classroom but that are not emerged by the use of ChatGPT.





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In Cabri we have the Grid tool, which provides a discrete set of points at which both coordinates are integers. This option then gives us the set of dots (p, q) that is required in the Babylonian method (Figure 10).



Fig. 10. The tool Grid for a discrete exploration of a property: Roberts' Theorem.

From here, with the Calculator tool we can generate the formulas for each number of the Pythagorean triple: $a = p^2 - q^2$, $b = 2 p \cdot q$, $c = p^2 + q^2$. The Calculator tool use the Basic language for its operation (Figure 11).



Fig. 11. The Babylonian method with the tools Grid and Calculator of Cabri II plus

A problem with Synthetic Geometry is to construct the triangle by knowing the measurements of its sides. In the case of Pythagorean triples, you can take advantage of creating the macro for the general case to create the right triangle that corresponds to the generated triple. Due to the dimensions of the triangles that can be obtained, it is convenient to create a button that hides or shows the right triangles that are obtained (Figure 12).



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When studying the relationship of the Babylonian method with the matrix product mentioned in Roberts' theorem, the Cabri Grid highlights the organization of the tree diagram in Figure 8: the vertical ray with the (1,2n) points contains ordered pairs that will generate Pythagorean triples identical to those obtained by multiplying $(3,4,5) \cdot U^n$. The ray (n,n+1), which is parallel to the right identity, contains the points with ordered pairs that create identical Pythagorean triples to be multiplied $(3,4,5) \cdot D^n$ (Figure 13).



Fig. 13. Bisector and Pythagorean triples obtained by $(3,4,5) \cdot A^n$ on Cabri II plus. Selective visualization with the Button Show / Hide tool.

The most complicated case is that of the pattern that follows multiplication $(3,4,5) \cdot A^n$ which does not follow a simple rectilinear trajectory. A line to compare in this situation is the bisector between the rays mentioned in the previous paragraph, which can also be built in this construc-





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tion. All 3 cases can be selectively grouped together to have selective visualizations and support student understanding in the classroom, using the Hide/Show Button tool (Figure 14).



Fig. 14. Exploration of Pythagorean triples, right triangles, according to the Babylonian method and Roberts Theorem with the interactive mathematics of Cabri II plus

In addition, the visualization of Dynamic Geometry offers us replicate the enthusiasm of the anonymous scribe on the Plimpton tablet 322 (Figure 5). We think in the future ChatGPT could offer an interactive environment to reproduce its linguistic descriptions or calculations, not only programs; actually, this approach is very useful in the classroom.

Conclusions and suggestions for the classroom

In the interaction with ChatGPT, version 3.5, these problems showed that the AI gives preference to searching the Internet for information posed with the purpose of solving mathematical problems, prioritizing numerical and symbolic calculations, even allowing the creation of program codes, which is in the guidelines for use in the classroom (Melchor, Lomibao & Parcutilo, 2023). However, in terms of timelines, sometimes it makes no distinction between historical stages, even falling into notable inconsistencies: in this case it is not clear how it focuses its searches.

The use of ChatGPT, version 3.5, had the advantage of offers a search tool, but sometimes it didn't go too deep into the subjects despite the advantages like immediate feedback, access to large amounts of information, helps to reduce the information search time, and also limitations like do not provide critical thinking or problem-solving skills, or its responses may be biased by the data set as referred by Green-Harper (2023).

Furthermore, with regard to geometric modeling, this is left aside by ChatGPT version 3.5. Perhaps due to a lack of bibliographic material at this address, ChatGPT should search for infor-



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mation such as that which appears in the manuals of graphing calculators, computing packages, or websites for the different Dynamic Geometry options. And again ChatGPT should not privilege what is recent, since there is material that has had a great impact on different academic communities.

Al learning is another point to improve (Belda, 2011): in Mathematics, conjectures are made based on observing regular patterns within a significant data set and machines have improved human performance in fields such as playing chess with techniques that the human brain cannot perform in decision trees. However, here we see a long path for Al to be able to generate or find patterns, establish and prove mathematical properties (Gadanidis, 2017). Al can be helpful in the construction of mathematical knowledge as proposed by the president of National Council of Teachers of Mathematics, Kevin Dykema⁶, but it will still need to include what the ancient Greeks identified in Pandora's behavior: human curiosity.

In the current classroom there is the expectation that it will become a bustling space where we actively work considering sources of the HM with the versatility of the tool that is AI, with a powerful computing machine and fountain of references in the hand. The expectation in the future for the education area is that open AI become more and more devices that learn, being able to generate or find patterns, relationships, regularities in several mathematical problems, because the society will require this kind of tools and professionals working with them. One example of that path, is DALL-E 3 that Open AI recently announced as a device to create AI images with ChatGPT (Gadanidis et al., 2024).

On the other hand, there is resistance in the classroom, both by teachers and students to use it in everyday work. However, actually it is growing the number of common citizens that use resources like apps to follow maps, to administrate banks accounts, and so on, to save time, increase their comfort and productivity. Then, why do not use an AI at school or in Math activities? Certainly, the geometric properties and its discoveries seems a challenge too big, but the references of Dynamic Geometry are very recent (at scale of HM).

ChatGPT did not manage "to sweep the corners of the libraries", then the teacher must be attentive to giving more expert instructions to collect information and observe the quality of what the student generates with any kind of tool.

For students, and from what has been shown, should be clear that the tasks solved with ChatGPT, version 3.5 do not always comply with the expectations of search or power in the calculation; This requires redoubled attention and more careful analysis of the information obtained with AI.

For teachers, it is not possible to escape the role of guardian of the quality and veracity of the information presented in the student projects that use AI. A priori, the projects and tasks to be developed must be structured in advance and with much more care by teachers, due to the elements that this resource provides and the need to complement it with other sources and forms of learning with existing technologies (dynamic geometry software, calculators with symbolic manipulation, and others).

For an educational action in the classroom it is important to gradually introduce this technology in the classroom, reporting the limitations that are found and the benefits that arise from its use. In this document HM was central for testing the use of IA and to highlight the need of awareness about

⁶ The complete message to the mathematical comunity is in the following link: https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Kevin-Dykema/Math-and-Artificial-Intelligence/



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the mentioned limitations and benefits. We should also note that the four historical examples we have discussed here show that MH can bring up open problems in Mathematics and this can test both the teacher and the students. This element must be handled properly in the school environment so that students do not lose motivation and that the teacher's experience is not undervalued.

In this text, we have seen that the four examples can help strengthen both the first university courses and the more advanced courses (the latter by virtue of the fact that it bring with it open problems). ChatGPT version 3.5 and other AI tools should be used to improve typing, support search power, and exploit the computational potential and depth of exploration of mathematical properties. ChatGPT version 3.5 (and his new versions) should not be overrated for school teaching. Traditional technology tools are valuable and powerful; adapt and create an e-symbiosis among both resources will be very useful to reorganize practices in mathematics classrooms in the transition to a new kind of learning in mathematics and science (Dykema⁶).

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