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Using an interdisciplinary approach to the teaching of solid geometry in a professional development course for preschool and primary school teachers

Uma abordagem interdisciplinar no ensino de geometria sólida num curso de formação contínua para educadores de infância e professores do 1º ciclo

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Abstract

This paper presents some results of a professional development course for teachers where the participants studied basic solid geometry and developed applied projects in an interdisciplinary context. The course took place in a Portuguese university, from February to May 2020, and involved 19 teachers of preschool and primary levels (grades 1 to 4).
The authors have developed a qualitative case study to evaluate how an interdisciplinary approach to the teaching of solid geometry is perceived, by the mathematics teachers, as a contribution to the teaching/learning process of geometry. Overall, the activities developed have proved to be successful examples of interdisciplinary methodologies. Moreover, the approach followed during the course helped the teachers develop their geometric competences concerning solid geometry in a more consistent appropriation and application of the geometric concepts involved.

Keywords: professional development; mathematics; solid geometry; mathematics education; interdisciplinarity.

Resumo

Neste artigo, são apresentados alguns resultados de uma ação de formação para professores. Nessa ação de formação, os formandos estudaram os sólidos geométricos e desenvolveram

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projetos num contexto de interdisciplinaridade. A ação de formação decorreu numa universidade portuguesa, entre fevereiro e maio de 2020. Envolveu 19 professores do ensino pré-escolar e do ensino básico.
As autoras desenvolveram um estudo de caso, com o objetivo de avaliar se uma estratégia interdisciplinar pode ser percecionada, pelos professores, como uma mais-valia no processo de ensino/aprendizagem da geometria. No geral, as atividades desenvolvidas evidenciaram ser bons exemplos de metodologias interdisciplinares. Além disso, a abordagem adotada durante a formação contribuiu para os formandos desenvolverem competências de geometria, nomeadamente, no que respeita a sólidos geométricos, numa mais sólida apropriação e aplicação dos conceitos envolvidos.

Palavras-chave: Formação contínua; matemática; geometria sólida; educação matemática; interdisciplinaridade.

Resumen

En este artículo se presentan algunos resultados de un curso de formación continua para profesores. En este curso, los alumnos estudiaron los sólidos geométricos y desarrollaron proyectos en un contexto interdisciplinario. Lo curso se llevó a cabo en una universidad portuguesa, entre febrero y mayo de 2020. En el participaron 19 profesores de educación infantil y primaria.
Los autores desarrollaron un estudio de caso, con el objetivo de evaluar si una estrategia interdisciplinaria puede ser percibida, por los docentes, como un valor agregado en el proceso de enseñanza-aprendizaje de la geometría. En general, las actividades desarrolladas resultaron ser buenos ejemplos de metodologías interdisciplinarias. Además, el enfoque adoptado durante la formación contribuyó a que los alumnos desarrollaran habilidades geométricas, es decir, en lo que respeta a los sólidos geométricos, en una apropiación y aplicación más sólida de los conceptos involucrados.

Palabras clave: Formación contínua; matemáticas; geometria solida; educación matemática; interdisciplinariedad.

Introduction

Nowadays, the inherent complexity of nature and society makes it impossible to restrict problems to a single area of knowledge. As it is necessary to fight the fragmentation of knowledge, it is essential to reflect on the role of the school, namely, about the use of interdisciplinarity in an educational context because, according to Morin (2003), the fragmentation of teaching will only serve to isolate objects from its environment and the parts of a whole. Also, according to this author, the disciplinary[1] nature of formal education[2] does not encourage the establishment

[1] In the context of this work, we consider a discipline as a curricular component, that is, as a way of organizing, delimiting, representing a selection of content to be explored according to previously defined objectives. (Morin, 2003).

[2] Gohn (2006), defines formal education as "that which is developed in schools, with previously defined content" (p.28).

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of connections between facts and concepts, nor the development of intelligence, making it difficult for the student to learn. The school must break with this atomization and try to show the correlations between knowledge, the complexity of current society and existing problems. Only in this way will it be able to train individuals prepared for the future. Pombo (2003) argues that interdisciplinarity is the way forward because it makes use of the values of convergence and complementarity. Also, Paviani (2008), defends interdisciplinarity as a solution to the excessive fragmentation of knowledge.

In the case of mathematics, one of the topics most challenging for teachers is geometry, as "it is one of the subjects in which students reveal a lot of difficulties" (Fernandes and Gomes, 2013). The same authors argue that the search for approaches that promote its learning in a motivating and meaningful way is pertinent.

Geometry related topics make a very good setting for an interdisciplinary approach including the arts and technology. One such topic is the study of solid geometry, which is present in the school mathematics curriculum in Portugal, from the early stages of preschool till the last year of secondary education (Bivar et al., 2013).

Professional development courses provide a means to disseminate new methodologies and teaching practices among in-service teachers. This paper presents some results of a professional development course for teachers where the participants studied solid geometry concepts and developed applied projects in an interdisciplinary context.

Theoretical framework

Professional development

Today's fast changing pace, allied to the complexity of mathematics teaching practices, places extraordinary demands on in-service teachers (Hall & Pais, 2018). Professional development is an important means to help teachers find ways to meet these demands. For almost thirty years, the mathematics education community has made a continuous effort to change teaching practices towards being more cognitively demanding, conceptually oriented and student centred (Heyd-Metzuyanim et al., 2016).

A lot of research on teacher's professional development has been done in the past years. Borko et al. (2010) highlight the recent shifts in professional development methodologies. These shifts are somewhat related to recent changes in the prominence of ideas about the nature of cognition, learning and teaching. As the authors refer, professional development should move away from a rigidly structured in-service training model towards approaches grounded in classroom practice and involving the formation of professional learning communities. Indeed, as Higgings and Parsons (2009) wrote, "professional learning opportunities for teachers need to be situated in the teacher's context of practice and relevant to the teaching and learning needs of teachers and students" (p. 232). These learning opportunities should focus on the real context of the classroom and be integrated into the teachers' everyday work.

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Interdisciplinarity

Interdisciplinarity started to be debated as a pedagogical concept in the second half of the last century (Pombo, 2003). Since then, many authors have tried to contribute to the discussion, but no common meaning has been yet accepted by the educational community (Paviani, 20084; Pombo, 2003; Zaman & Goschin, 2010).

In the context of this work, the definition of interdisciplinarity in Palmade's perspective was adopted. The author proposes that "interdisciplinarity should be defined as 'the internal and conceptual integration that breaks the structure of every discipline, building a new axiomatic, common to all of them, with the aim of giving a unitary vision of a sector of knowledge '" (Palmade, 1979, as cited by Pombo, 1993, p.10).

One way to think about teaching using an interdisciplinary approach is through STEAM education. This methodology proposes the integrated teaching of science, technology, engineering, arts and mathematics. According to Shaughnessy (2013), "STEM education refers to the solution of problems that take advantage of mathematics and science concepts and procedures, incorporating group work, methodologies and engineering design, with appropriation of technology" (as cited by English, 2017, p. 5). The addition of the arts to STEM education, changing the designation to STEAM, expanded the scope of the methodology. STEAM education corroborates Jackson (2011) when he reports as the first top lesson Steve Jobs taught us: "the most enduring innovations marry arts and science". This lesson shows that the incorporation of the arts is fundamental in the globalized context of current societies.

In this perspective, it is impossible to think about STEAM Education without considering the context of interdisciplinary practices. "On the contrary, the two proposals complement each other in the construction of sequences of interdisciplinary teaching focused on five major areas of knowledge, promoting the theoretical and practical development of the student" (Machado & Junior, 2019, p. 52). In early ages such as preschool and primary grades, STEAM education can only be addressed in a very embryonic stage. In this sense, interdisciplinarity presents itself as a more adequate framework for the purpose of the present work. In addition, interdisciplinarity encompasses a wider range of combinations, far beyond the five areas assigned to STEAM education.

Regarding the teaching of geometry, there are many topics that can be related to other disciplines, which makes it possible to approach it from an interdisciplinary perspective. Based on authors such as Paviani (2008), Zaman and Goschin (2010), among others, we believe we can affirm that the adoption of an interdisciplinary approach in teaching, namely in geometry, is an effective strategy to improve teaching and learning, because when contextualizing with other areas of knowledge, students realize that such a subject is in fact important both for their day-to-day life and for addressing other subjects.

Geometry in Education

Geometry, "considered as the science of space" (Gordo, 1993, p. 35), is present in different situations of our day-to-day, and allows us to develop skills to know "to describe, analyze

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and understand the world that surrounds us" (Brocardo & Mendes, 2007, p. 7). Its teaching is therefore fundamental.

Traditionally, the teaching of geometry is still very much focused on a symbolic and algebraic methodology. In what concerns the learning of geometry contents, Matos and Serrazina (1996) state that it is essential to resort to "a methodology that starts from the student's vision and that provides him with the means and the environment so that he himself develops his knowledge" (p. 265). Walker et al. (2011), argue that the dual perspectives of formal symbolism and visualization are complementary approaches to conceptualizing the same geometric task. In this sense, the teacher must adopt a didactic approach that uses the manipulation of objects and materials, in the initial phases of learning, and later, resort to more abstract tasks, which appeal to the student's spatial visualization ability (Brocardo & Mendes, 2007).

According to Walker et al., (2011, pp. 22-23), "we have begun to explore the possibility that the development (and exercise) of visualization skills in non-mathematical domains — for example, the visual arts — may support students' geometric thinking". The same authors proceed by arguing that, "it is possible, therefore, that students who acquire training in the visual arts may be able to apply their visualization skills to support their mathematical and scientific thinking as well" (2011, p. 23).

Solid Geometry

A solid can be defined as an enclosed portion of space, bounded by plane and curved surfaces (Rich & Schmidt, 1989). Solids may be classified according to their characteristics and some families of solids are commonly addressed in school curricula:

1. Polyhedra
   a. Prisms
   b. Pyramids
c. Platonic solids
2. Cylinders
3. Cones
4. Spheres

All solids have three dimensions, and they are useful to model real world objects. Practical illustrations of solids include a box, a ball, a dice, and an ice-cream cone. In geometry, we are interested in the geometric properties of solids such as their volume or the area of their surface.

Solid geometry is addressed throughout all the school curricula, starting as early as kindergarten. Children are progressively introduced to geometric solids of the above-mentioned families, starting with the most common and simple solids, such as the cube, the sphere, the circular straight cylinder and cone, and the pyramids. Further on, they learn about other members of these families, including straight and oblique solids, convex and concave solids (Bivar et al., 2013).

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To study solid geometry, a previous or parallel study of plane geometry must occur. Students learn about areas and perimeters of plane figures and apply these concepts to solids. Calculating the volume of solids is one of the final steps in the study of solid geometry during school years. Along the curricular path, another important aspect is addressed: the net of a solid, that is, a 2D shape that can be folded to form the solid. Through nets, students can make manipulative models of solids that help them understand their characteristics. They also help develop student's spatial visualization skills which are fundamental not only in geometry but in general problem solving (Walker et al., 2011; Veloso, 2011).

Method

Methodological options

The research question underlying this study is:

How is an interdisciplinary approach to the teaching of solid geometry perceived by teachers as a contribution to the teaching/learning process?

As a consequence, the following objectives were defined.

Investigate if an interdisciplinary approach contributes to:

- a more solid appropriation and application of the geometric concepts involved;
- making the learning process more interesting;
- promoting a more positive attitude towards mathematics in general and geometry in particular.

In this perspective, a qualitative case study was developed, grounded on a pragmatic paradigm and case study design (Carmo & Ferreira, 1998; Ponte, 2006; Yin, 1994). The study was undertaken in a Portuguese higher education institution involving the participants of a professional development course intended for preschool and primary school teachers. The teacher responsible for the course in question is simultaneously a researcher of this study.

In order to develop this experience, the techniques of inquiry, direct observation and documental collection were applied, and the following instruments were used: field notes, participants' reports and final questionnaire.

Description of the course

The professional development course addressed in this paper is part of a set of professional development courses for mathematics teachers promoted over the last years following the ideas presented in the first section of this paper. In these courses, we try to integrate the activities into the teachers' everyday work, while at the same time provide an opportunity to both learn and have a fulfilling and gratifying experience. Following the ideas presented by Borko et al. (2010, p. 548),

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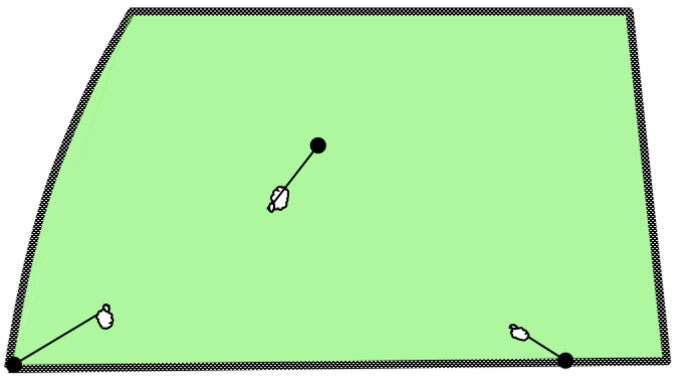
these courses are intended to be "opportunities grounded in a conception of learning to teach as a lifelong endeavour", that should be both pleasurable and rewarding. At the same time, we promote the integration of diverse areas of knowledge and move towards Eisner's conception of a practice rooted in the arts (Eisner, 2002): teachers take on the role of an artist: they are given time to explore, to create and to surprise themselves.

The professional development course in focus was titled "Sólidos a torto e a direito" (Solid figures in many ways) and took place in the University of Aveiro, in 2020, from February 5th to May 30th. Like all professional development courses for Portuguese teachers, it was acknowledged by the national scientific and pedagogical council for teacher's professional development (Conselho Científico-Pedagógico da Formação Contínua) and was registered with the number CCPFC/ACC - 104046/19. This course consisted of 25 hours of contact between all participants. Due to the occurrence of the Covid19 pandemic, the last session was online using the Zoom application. The course had 19 participating teachers, nine of which in preschools and ten in primary schools (grades 1 to 4). All participants were female teachers and had a significant teaching experience: 13 had more than 20 years of teaching, two between 10 and 20 years and the others between 5 and 10 years. No one had less than 5 years of teaching practice.

In the first part of the course (10 hours), some concepts were provided or recalled and related activities were carried out. The participants performed several tasks intended to deepen their knowledge on geometry. The first set of tasks focused on plane geometry (nine tasks), recalling some usual plane figures (circles and polygons) and their characteristic (length of sides, area, perimeter). The second and main set of tasks focused on solid figures (18 tasks) exploring some of their characteristics (family characteristics, nets, volumes, Euler's formula for polyhedra). Participants recalled important characteristics of some well-known families of solids (prisms, pyramids, cylinders, cones and spheres) and learnt about other less usual families (the Platonic solids and Archimedean solids). Participants had to draw and cut several nets to answer a diversified set of challenges concerning geometric solids.

Some examples of the proposed tasks are given next:

- In a meadow full of fresh grass, there are several sheep grazing. Each sheep is attached to a post by a relatively short rope and cannot move freely. Paint with a brown or black pencil the area of the meadow that will be clear of grass at the end of the day.



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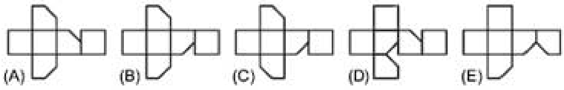
- If we cut out one of a cube vertices, which of the following nets do we get?



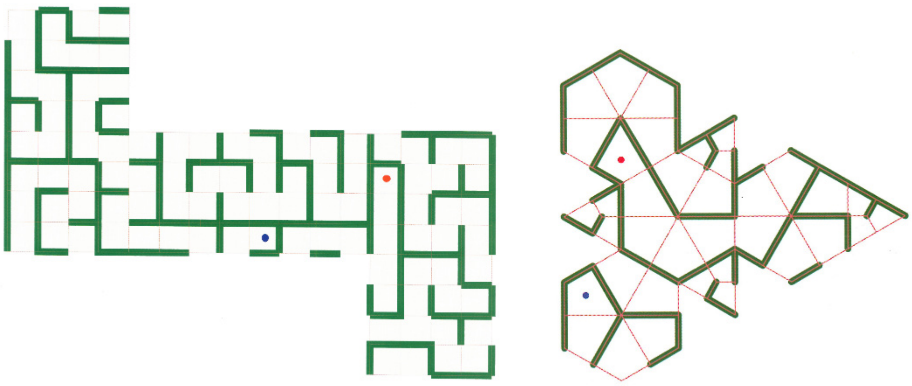
Draw the net of two square pyramids, one right and the other oblique. Cut and fold your nets to check out the result.

Using an isometric grid (of equilateral triangles) draw several configurations of four adjacent triangles and identify those that are tetrahedron nets.

Cut the platonic solids nets retrieved from Thomas (2019, pp. 589). Fold them, punch small holes where signalled and pass a thread through the holes of each one. Pull the threads to pull-up the solids.

Cut out the nets of the 3D mazes that follow. Do not forget to leave margins to glue adjacent faces. Fold and glue. Can you identify the solids?

Two of the nets are given next:



- Prepare two equal-sized parallelepipedal cardboard boxes (e.g. cereal boxes). Make a pyramid from one of them by choosing one face as base and maintaining the same height as the other box. Leave one face open in both boxes. Using rice, check that 3 pyramid measurements are needed to fill the other box.

In the second part of the course (15 hours), the participants were asked to develop a project (individually or in group) that put the theoretical concepts into practice, in an interdisciplinary context. The participants were challenged to include an artistic dimension to their projects, and to involve their students in the project. All participants were asked to present their project to the group in the final session of the course.

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According to the initial plan, these 15 hours were to be divided into three face-to-face sessions: two sessions of practical work for the making of the projects (one full day (7,5 hours) plus half a day (4 hours)), and one last session (3,5 hours) to make a class presentation of all the projects. Due to the Covid pandemic, only the first 7,5 hours were face-to-face. In this session participants brought their project plans and materials and worked hands-on them. Some weeks after this session a national lockdown was imposed. The remaining 7,5 hours had to move to an online regime. All participants intended to involve their students in the making of the projects. However, some did not have the opportunity do this and others could not complete it. Therefore, many participants had to finish their projects by themselves, at home. Despite all the difficulties, one of the participants, managed to engage her preschool students online on the making of her project!

The online hours of the course (7,5 hours) were used to make presentations and discussions of the projects between all participants.

Results

Participants' outcomes

All participants responded very actively and creatively to all the proposed activities, including the projects of the second part of the course. All projects stemmed from solid geometry, merged an artistic dimension and used interdisciplinarity, as proposed. They involved the construction of solids, starting with calculations and planning of the nets, and proceeding to the hand-on construction using raw materials (such as paper, cardboard, fabrics and recycled stuff). In the end all solids were finished with artistic effects (using paint and other materials with diversified textures and colours).

Many projects linked mathematics with the Portuguese language. Some projects combined environmental concerns or natural sciences topics while others had a clear urban setting. One of the projects involved robots (including rudimentary programming tasks and algorithm tackling). Table 1 resumes the most important areas interlaced in the projects.

Table 1: Summary of projects

Project	Title/brief description	Areas (all projects include solid geometry and artistic expressions)
Proj.1*	*Casa da música* (concert hall) with surrounding gardens	architecture, cultural heritage, garden planning
Proj.2	*"Who wants to marry the little sphere?"* Adaptation of a traditional Portuguese story (involving animals) to the context of geometric solids	Portuguese language (including storytelling), theatre, nutrition
Proj.3	Replica of a *Costa Nova* house	Architecture (3D modelling and 2D blueprints), local cultural heritage; house furniture and decoration.
Proj.4*	From the city of solids into space	House construction, astronomy (space shuttles, planets and stars).

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Proj.5	Construction of a city of solids	City planning, garden planning, environment.
Proj.6*	Forest of solids	nature protection and preservation; Portuguese language (including storytelling).
Proj.7*	Robot project	Robotics and programming, city planning and environment.
Proj.8*	Solids out of the box	Portuguese language (including poetry and riddles), theatre.
Proj.9	Leopoldina and Rita – Construction of two story-telling puppets (a bird and a girl) out of geometric solids	Portuguese language (including storytelling), theatre.
Proj.10	Platonic solids – Exploration of a children's book through the five symbols of the platonic solids (water, fire, earth, air and universe) and creation of a board game with question cards	History, sciences (water cycle, geography, air pollution, space and planets, environment …), Portuguese language (including storytelling).

*project described below in more detail

Next, a description of a selection of the projects developed is given.

Casa da Música

In Oporto, Portugal, the most important concert hall is named *Casa da Música* (music house). Completed in 2005, it was designed by the Dutch architect Rem Koolhaas and its shape follows a highly irregular polyhedron with 16 sides. Because of the uniqueness of its shape the concert hall is visited by students and tourists just for its architecture.

One of the participants of the course had a visit to *Casa da Música* planned with her preschool children. She decided to develop her project within this context. The group made a carton "replica" of the *Casa da Música"* surrounded by special ornamental trees (see Figure 1). These trees had their canopies shaped as platonic solids. Children were given the nets of the solids and had to decorate and fold them. A net of the concert hall may be found in https://architectures.jidipi.com/a417207/casa-da-musica/. The high regularity of the trees contrasts with the irregularity of the concert hall.

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Figure 1: Casa da Música (preschool teacher)

From the city of solids into space

One of the preschool teachers participating in the course developed with her children a set of geometric solids which could be used for different purposes. They used them to build a city and a set of space-shuttles. They used cubes, triangular prisms, square pyramids, cones and cylinders as their building blocks. The results may be seen in Figures 2 and 3.

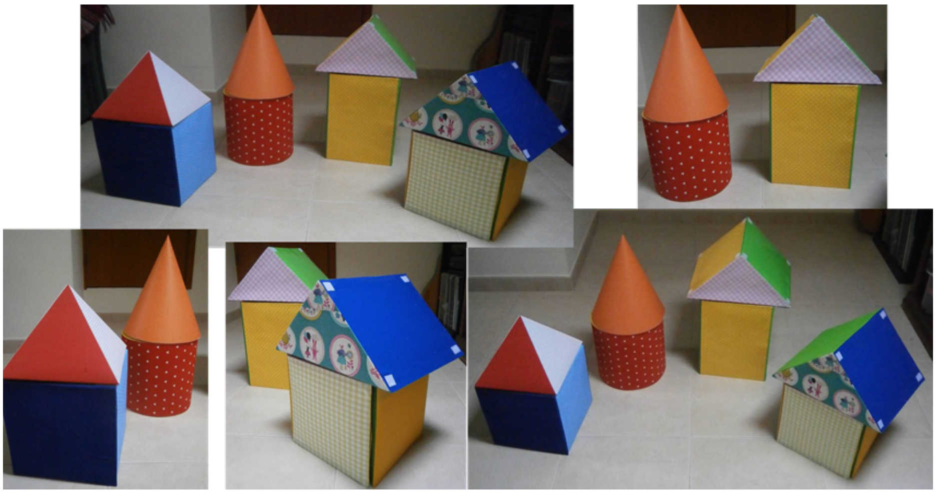

Figure 2: City of solids (preschool teacher)

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Figure 3: Space-shuttles (preschool teacher)

Building a city with different geometric solids was also done by other teachers. All the projects developed provide important opportunities to explore geometric concepts and spatial awareness with the children. One teacher specifically enumerated some of the learning outcomes that can be obtained through the exploration of the solids used:

> "With the exploration of the *City of geometric solids* we can explore different concepts, such as:
> distinction between polyhedra and non-polyhedra;
> - elements of polyhedra;
> - distinction between prisms and pyramids;
> - geometric figures;
> - itineraries."

The forest of solids

A group of three preschool teachers undertook a project integrated with environmental awareness and protection of the natural world, in particular forests. Their project consisted of creating a special forest and inventing a story integrating all the issues involved: solid geometry, storytelling, and preservation of forests. The story included a description and naming of the solids that modelled the trees. The trees were made collaboratively by the teachers and children. In addition to the forest and story, the group also created several replicas in a small scale of the solids involved and made a game of identification of solids. Children had to place the smaller solids on the trees with the corresponding shape. Figure 4 shows the forest and some photos of the children playing the game.

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Figure 4: The forest of solids (preschool teachers)

As happened in the previously mentioned project of the city of solids, this group of teachers reported the learning opportunities for their children, in the context of geometry, that emerged during (and after) the construction of the forest. Using different materials, children explored the geometric flat figures that appeared in the solids (triangles, squares, rectangles and circles). Next, they built several solids, firstly using plasticine (to explore and compare characteristics), and secondly using cardboard for the forest itself. The teachers said that this hands-on process was a powerful way to give meaning to their spatial sense, and an opportunity to explore geometrical concepts and properties (vertices, faces, edges, …). The game that they built provided an excellent opportunity to practice the identification of geometric solids, in a game-based context.

Solids out of the box

A group of three primary teachers decided to interlace the study of geometric solids with the Portuguese language. They wrote a set of rhymes for describing the solids. The rhymes have a plot built around a contest to decide which of the solids should be elected as the best one. In the end, they were all tied since they were all considered equally important.

The group of teachers built a set of solids, mimicking cute heads, and kept them together with the rhyme cards in a nicely decorated cubic box (see Figure 5). They called it “Solids out of the box”. They also attached a set of activities to be performed after the parade of rhymes. These include:

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Identification of solids through the rhymes; description of solids, considering their characteristics, without saying your name; filling up a table with the solid's characteristics; Solid construction with various materials (toothpicks, plasticine ...); Solid construction from nets; Identification of real-life objects illustrating geometric solids; Elaboration of riddles, crosswords and word search puzzles; Creation of an oral or written story, its dramatization, choreography and song writing.

Figure 5: Solids out of the box (primary school teachers)

Robot project

Our final example comes from a group of four primary school teachers, who developed an interdisciplinary project linking the study of the environment with mathematics, robotics, and the arts. Geometric solids were used to model several city buildings (such as the hospital, the school, the library and the town hall). A large scenery base, used as a city plan, was created through a square grid over which a robot would circulate (see Figure 6). The solids were displayed on some of the grid cells and several circuits were defined. The students had to program the robot so that it would follow the steps given in each circuit description. The robot used was a *blue-bot* and this activity was integrated into an interinstitutional project on robotics, that the school had with the University of Aveiro and the Intermunicipal Community of the Region of Aveiro.

The teachers involved in this project began their report with the following self-explanatory introduction:

> This work aims to exemplify the possibility of articulating several areas of knowledge and propose meaningful learning outcomes in a different (less common) way, which is dynamic, challenging and playful (game-based learning).
> Thus, using the areas of artistic expressions, study of the environment, mathematics, programming and robotics, we developed a set of solids with which we built some buildings that have been identified with some real institutions and services[3].

[3] e.g. library, school, hospital, supermarket, police station, fire station.

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It should be noted that, for now, we only intend to give an example that later, with the children, can be explored, deepened and complexified.[4]

This type of proposal has applications in all areas: Portuguese language (phrase construction to make the robot move according to instructions; riddles, rhymes, ...); mathematics (itineraries, operations, mental calculation, etc.); study of the environment (cities, paths, etc.)

Imagination and challenge will be the key to all possible choices.

Figure 6: Robot project (primary school teachers)

An example of the robot following one of the circuits can be seen in youtube[5] (one can see the robot being programmed and following the given instructions).

Other results

This paper intends to analyse how an interdisciplinary approach to the teaching of solid geometry is perceived, by teachers, as a contribution to the teaching/learning process. As a consequence, three objectives were defined and stated previously in the method section.

To reach this goal, direct observation, field notes and analysis of the reports that accompanied the final presentations were the main instruments used. In addition, a questionnaire was answered by the participants at the end of the course.

[4] due to the Covid pandemic, the teachers could not explore the final project with their students.

[5] https://youtu.be/k4PP4IBWNwY

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Regarding the first objective stated previously – investigate if an interdisciplinary approach contributes to a more solid appropriation and application of the geometric concepts involved – several results may be reported.

During the first part of the course, teachers had to perform a rather extensive set of tasks. They solved their tasks in a dynamic and participative way, sharing their difficulties and achievements. Through direct observation and field notes, the researchers could acknowledge that the participants were deepening their previous knowledge and acquiring new one. Later, during the course, the researchers witnessed a solid appropriation and application of solid geometry concepts during the project phase of the course. Participants managed to plan and construct their projects in an autonomous and fruitful way, applying concepts which had been explored previously. This conclusion is further reinforced by the participant's reports. A group of participants mentioned the importance of having learned about new families of solids, such as the Platonic and the Archimedean solids. Another group of teachers mentioned that materials such as *polydron* were unknown to them and revealed to be an excellent resource for their daily practice. This group specifically highlighted "the practical approach used and instilled during the course, which made the sessions dynamic and elucidative". They further stated they were "able to assimilate new knowledge and approaches, which enriched their work in the classroom". They also said that "the course was important for our practice, since it provided us with new knowledge, always within a practical and visual approach."

Through direct observation, it can be concluded that the participants were deeply interested in performing the tasks, including the final applied projects. The fact that they had a concrete goal to achieve – the development of an applied project within their teaching context – was an important source of motivation for and engagement with all the tasks. As the tasks were performed, we observed an increasing willingness to perform their own projects. These facts, together with the previous paragraph statements, support a positive answer to the other two established objectives: investigate if an interdisciplinary approach contributes to: making the learning process more interesting, and promoting a more positive attitude towards mathematics in general and geometry in particular.

Next, an analysis of the answers to the questionary delivered at the end of the course is provided. The questionnaire had four sections: a first section concerning the characterization of the respondent; a second section concerning a general evaluation of the course; a third section concerning the use of an interdisciplinary approach in the teaching of solid geometry/mathematics and a last section concerning the use of creativity in the teaching of mathematics.

Section 1: Participant characterization

The questionnaire was answered by 18 (out of 19) respondents. Eight respondents were preschool teachers and the remaining 10 were primary school teachers.

Section 2: General evaluation of the course

Teachers were asked to rate, through a five-point Likert scale (1-very negative; 5-very positive), their general evaluation of the course. Next, they were asked to give, through a five-point

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Likert scale (1-not at all; 5-very much), their agreement with a set of statements regarding some general aspects. Figure 7 shows the responses to these questions through a heatmap.

	1	2	3	4	5
What is your global evaluation of the course?	0	0	0	4	14
The topics of the course were relevant.	0	0	0	5	13
The topics of the course met my expectations.	0	0	1	3	14
The activities were interesting.	0	0	0	3	15
The objectives of the course were met.	0	0	0	4	14
The course updated my knowledge on the subject.	0	0	1	2	15

Figure 7: Heatmap of responses to the general aspects section.

As can be seen in the heatmap of Figure 7 there are no negative answers (levels 1 or 2) and only two neutral opinions. In all questions/statements, the majority of responses fell on the highest level of agreement, with at least 13 (72%) responses each.

- Teachers were also asked to name 3 weaknesses and three strengths of the course.
- The most frequent strengths stated were:
- The possibility to recall and deepen the knowledge about solid geometry.
- The clarity, objectiveness and accessibility of the course supervisor.
- The applied nature of the course, linking theory and practice.
- The diversity of experiences/activities during the course.
- The sharing of experiences/activities/information.
- New perspectives/approaches to the teaching of solid geometry.

There were very few weaknesses stated, most of which related to the pandemic since it did not allow some of the projects to be finished in the classroom with the students and the last session of the course had to be online.

Section 3: Interdisciplinarity in the teaching of mathematics

Regarding the interdisciplinary approach followed during the course, teachers were asked to answer the following Yes/No question: “*Did you appreciate the interdisciplinary approach used in this course to explore the geometric concepts involved?*”

Seventeen teachers replied “Yes” and one teacher did not answer. There were no negative answers.

Teachers were also asked to rate their agreement, on a five-point Likert scale (1-total disagreement; 5-total agreement), over a set of statements. Figure 8 shows the responses to these questions.

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The use of an interdisciplinary approach during this course…					
	1	2	3	4	5
contributed to a more positive view of mathematics	0	0	2	4	12
made learning more stimulating.	0	0	0	3	15
contributed to better understand the importance of mathematics.	0	0	2	3	13
provided a more interesting learning method.	0	0	0	4	14
contributed to the sessions being more dynamic.	0	0	0	3	15
broadened my horizons.	0	0	1	2	15
helped me to better understand the existence of relationships between mathematics and everyday life.	0	0	2	5	11
helped me establish an articulation between theory and practice.	0	0	1	5	12

Figure 8: Heatmap of responses concerning the use of an interdisciplinary approach in the course.

Still relating to interdisciplinarity, teachers were asked to rate, through a five-point Likert scale (1-not at all; 5-very much), their general I opinion about the use of interdisciplinarity in mathematics classes. The responses are shown in Figure 9.

	1	2	3	4	5
Do you find it interesting to use an interdisciplinary approach in the teaching of mathematics?	0	0	0	1	17
Do you find it important to use an interdisciplinary approach in the teaching of mathematics?	0	0	0	1	17

Figure 9: Heatmap of responses to the general questions concerning interdisciplinarity.

As can be seen in the heatmaps of Figures 8 and 9, there were no negative opinions concerning the use of interdisciplinarity during the course or in the teaching of mathematics, in general. In all questions, the most frequent answer was again in the highest level (5). Only four statements received one or two neutral responses.

Teachers were asked to give their opinion (open answer) to the question *"What opinion do you have regarding an interdisciplinary approach in the teaching/learning process of Mathematics?"* All the 12 responses revealed it is important. We present a selection of the answers which we consider most relevant (in brackets we give the number of responses with similar meaning):

- Mathematics is present everywhere, even in the very air we breathe. Long live interdisciplinarity! " (4x)
- It is important for students to better understand the applicability of mathematics to other subjects and its relationship with our daily lives. (2x)
- In the case of preschool, it is the only possible approach, which is, fortunately, being appreciated and reinforced in other levels of teaching. (2x)

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- As a primary school teacher, this is a common practice. Of course, I think this approach is very important and it meets what is called for in curriculum flexibility. (2x)
- Very important and indispensable since preschool education, for the integral evolution of children in future learning.
- It is always useful / interesting for us and the students.

Section 4: Creativity in the teaching of mathematics

Finally, teachers were asked to give their opinion concerning the use of creativity in mathematics teaching, by rating in a five-point Likert scale their agreement with two statements (1-total disagreement; 5-total agreement). Results are shown in Figure 10.

	1	2	3	4	5
I consider it important to use creativity in teaching mathematics.	0	0	0	0	18
The tasks/activities proposed in this course stimulated my creativity.	0	0	0	2	16

Figure 10: Heatmap of responses to the questions concerning creativity in the teaching of mathematics.

Like in the previous sections, there was a clear positive opinion regarding the use of creativity in mathematics teaching. All respondents totally agree that it is important to use creativity and all but two totally agree that the course stimulated their creativity. The other two respondents also agree (though not totally) with this last statement.

Final remarks

From the analysis of the questionnaire, it can be concluded that there was a positive influence of the interdisciplinary approach used in the course towards a more solid appropriation and application of the geometric concepts involved. Furthermore, the questionnaire reveals that creativity is also an important aspect in the teaching of mathematics. The teaching/learning process is a complex one and must combine different methodologies and approaches. Interdisciplinarity and creativity are some of the possible resources that may contribute to improve the process.

The obtained results are consistent with the filed notes registered by the teacher during the professional development course and with the reports handed in by the participants in the last session. Not only the participants' curiosity, motivation and enthusiasm were notorious and reliable but also their performance regarding both the preparatory geometry tasks and the applied projects revealed a suitable appropriation of the concepts involved.

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The use of an interdisciplinary approach during this course was facilitated by the teaching contexts of the participants: the age range of their pupils (3 to 9 years) naturally embraces interdisciplinarity at school. However, the unexpected appearance of Covid19 in March made it impossible for most of the projects to be concluded in the participants' classrooms. This was a demotivating factor for the participants which could not be overcome. In addition, the fact that the last session and corresponding presentation of the final projects amongst participants had to be online was also a drawback on the course. Nevertheless, all participants engaged as much as possible to concluding their projects and making the best presentations possible, recurring to videos and other technological tools for their final presentations.

The professional development course described in this paper aimed to investigate how an interdisciplinary approach to the teaching of solid geometry is perceived by the mathematics teachers, as a contribution to the teaching/learning process.

The participants showed an enormous commitment to the accomplishment of the proposed tasks and projects. Their performance throughout the entire course, including the reports handed in, showed that they improved their knowledge concerning solid geometry.

Overall, the activities developed have proved to be successful examples of interdisciplinary methodologies that bring into the teaching of mathematics, not only other areas of knowledge such as the Portuguese language, environment sustainability, robotics, but also usual procedures in the teaching of the arts, associated to the process of artistic creation. We believe that these teachers have been strengthened in their capacity to develop multidisciplinary tasks and projects with their students. The methodologies used, involving the interconnection between different areas, promote a positive attitude towards mathematics and thus foster the motivation to learn it.

The phase of development of interdisciplinary projects was particularly rewarding, since the participants were involved in every step of the process, dedicating all the necessary time to its accomplishment. Most of all, the participants showed enthusiasm, commitment and pleasure during the course and these are the main ingredients for successful teaching and learning.

The results of the present research show that the selected approach is positively perceived, by the participants, as a contribution to the teaching and learning process of solid geometry. The questionnaire filled out at the end of the course showed that the overall evaluation of the course was very positive and allows us to conclude that its goals were accomplished.

Overall, it can be concluded that all objectives established for this research were positively met since:

- the interdisciplinary approach followed during the course helped the teachers develop their geometric competences concerning solid geometry,
- from what the researchers observed during the course, the participants were interested and motivated, which is also reinforced by the analysis of the answers to section 2 and 3 of the applied questionnaire, namely in the first question of section 2,
- from the analysis of the applied questionnaire, it is possible to verify that the participants took a more positive attitude towards mathematics in general and geometry in particular.

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Although the objectives established were confined to the participants of the course, it is of interest to wonder if the interdisciplinary approach driven by the course also had a positive effect on the learning process of the participant's students. Several comments stated by the participants in their reports, some of which mentioned in the results section, point in this direction. One particular participant made the following comments:

The exploration of geometry tasks in a multidisciplinary context with preschool children shows that:

- It is possible to explore geometry with challenging and ludic activities.
- The tasks help acquire new competencies and positive attitudes towards mathematics, since they promote confidence and interest.
- Geometry is yet a further tool to develop creativity, visual and aesthetic senses, and intuition, in an integrated way.
- With geometry children can play and simultaneously acquire specific vocabulary, learn about plane figures and geometric solids, topological relations and spatial orientation."

As a final conclusion and regarding the research question - How is an interdisciplinary approach to the teaching of solid geometry perceived by teachers as a contribution to the teaching/learning process? – this study suggests that the teachers perceived the contribution as a positive one.

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